MATH 31 – HOMEWORK 4

due Wednesday, July 26

Instructions: This assignment is due at the *beginning* of class. Staple your work together (do not just fold over the corner). Please write the questions in the correct order. If I cannot read your handwriting, you won't receive full credit.

1. (8.4 and 8.10) *(8 points)*
   (a) *(3 points)* Let $\pi = (x_1\ x_2\ \cdots\ x_r) \in S_n$. Show that $o(\pi) = r$. (Note: this means implicitly that all the x_i are different.)
   (b) *(3 points)* Suppose that a permutation π is the product of disjoint cycles π_1, π_2, ..., π_m. Show that $o(\pi)$ is the least common multiple of $\{o(\pi_1), o(\pi_2), \ldots, o(\pi_m)\}$.
   (c) *(2 points)* Find the order of the permutation
   $$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 \\ 6 & 7 & 5 & 9 & 8 & 4 & 11 & 3 & 1 & 12 & 2 & 10 \end{pmatrix}$$
   in S_{12}.
2. *(4 points)* (8.12) Does A_6 have an element of order 6? Does A_7?
3. *(4 points)* Find the right cosets of the subgroup $H = \langle (1,1) \rangle$ in $\mathbb{Z}_4 \times \mathbb{Z}_4$.
4. *(4 points)* (9.13) Suppose G is a group and A and B are subgroups of G. Define a relation R on G by
   $$x\,R\,y \text{ if and only if there exist } a \in A \text{ and } b \in B \text{ such that } x = ayb.$$
   Prove that R is an equivalence relation on G.