

MATH 31 – HOMEWORK 3

due Wednesday, July 19

Instructions: This assignment is due at the *beginning* of class. Staple your work together (do not just fold over the corner). Please write the questions in the correct order. If I cannot read your handwriting, you won't receive full credit.

1. (6 points) (6.12) Let G and H be finite groups. Show that if $G \times H$ is cyclic, then (i) G and H are cyclic, and (ii) every subgroup of $G \times H$ is of the form $A \times B$ for some subgroups A and B of G and H , respectively.
2. (6 points) (6.13) Prove the converse of the result in the previous question. That is, show that for finite groups G and H , (i) and (ii) of the previous question (taken together) imply that $G \times H$ is cyclic.
3. (4 points) Write each permutation as a product of disjoint cycles, then as a product of transpositions. Determine whether each permutation is even or odd.

(a) $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 2 & 4 & 6 & 1 & 3 & 5 \end{pmatrix}$

(b) $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 6 & 5 & 4 & 1 & 2 & 3 \end{pmatrix}$

(c) $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 3 & 7 & 1 & 5 & 6 & 4 & 2 & 8 \end{pmatrix}$

(d) $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 8 & 1 & 2 & 3 & 7 & 4 & 5 & 6 \end{pmatrix}$

4. (4 points) (8.7) Prove that S_n is nonabelian if $n \geq 3$.