

MATH 31 – HOMEWORK 2

due Wednesday, July 12

Instructions: This assignment is due at the *beginning* of class. Staple your work together (do not just fold over the corner). Please write the questions in the correct order. If I cannot read your handwriting, you won't receive full credit.

- (4.13) Show that if G is a finite group, then every element of G is of finite order.
 - (4.14) Give an example of an infinite group G such that every element of G has finite order.
- Find all subgroups of $(\mathbb{Z}_{30}, \oplus)$, and draw the subgroup lattice.
- (5.24) *This exercise proves a condition that helps you test if a subset is a subgroup.* Let G be a group and let H be a nonempty subset of G such that whenever $x, y \in H$ we have $xy^{-1} \in H$. Prove that H is a subgroup of G .
- Let G be an abelian group and suppose that G has at least two distinct elements of order 2. Show that G has a subgroup of order 4.