MATH 31 – HOMEWORK 1

due Wednesday, July 5

Instructions: This assignment is due at the *beginning* of class. Staple your work together (do not just fold over the corner). Please write the questions in the correct order. If I cannot read your handwriting, you won't receive full credit.

1. (3.11) Let $(G, \star)$ be a group such that $x^2 = e$ for all $x \in G$. Show that $(G, \star)$ is abelian. (Here, x^2 is shorthand for $x \star x$.)
2. (3.12) Let $(G, \star)$ be a group. Show that $(G, \star)$ is abelian if and only if $(x \star y)^2 = x^2 \star y^2$ for all $x, y \in G$.
3. Let $G = \{5, 15, 25, 35\}$ and let $\star$ be the operation of multiplication modulo 40. (For example, $15 \star 35 = 5$.) Show that $(G, \star)$ is a group.
4. For any two sets A and B, the *symmetric difference* $A \triangle B$ is defined by

   $$A \triangle B = (A \setminus B) \cup (B \setminus A).$$

   In other words, the symmetric difference is the set of elements that are in A or in B but not in both.

   The *powerset* of a set S, denoted $\mathcal{P}(S)$ is the set of all subsets of S, including the empty set and the full set. For example,

   $$\mathcal{P}(\{1,2,3\}) = \{\emptyset, \{1\}, \{2\}, \{3\}, \{1,2\}, \{1,3\}, \{2,3\}, \{1,2,3\}\}.$$

   Fix a set S. Prove that $(\mathcal{P}(S), \triangle)$ is a group.