NAME : ____________________

Math 31

Midterm 1
July 12, 2017

Prof. Pantone

INSTRUCTIONS: This is a closed book exam and no notes are allowed. You are not to provide or receive help from any outside source during the exam except that you may ask the instructor for clarification of a problem. You have 120 minutes and you should attempt all problems.

- Print your name in the space provided.
- Calculators or other computing devices are not allowed.
- Except when indicated, you must show all work and give justification for your answer. **A correct answer with incorrect work will be considered wrong.**

All work on this exam should be completed in accordance with the Dartmouth Academic Honor Principle.

TIPS:

- You don't have numerically expand all answers.
- Use scratch paper to figure out your answers and proofs before writing them on your exam.
- Work cleanly and neatly; this makes it easier to give partial credit.

Problem	Points	Score
1	20	
2	20	
3	15	
4	15	
5	15	
6	15	
Total	100	

Section 1: True/False.

1. (20) Choose the correct answer. *No justification is required for your answers. No partial credit will be awarded.*

(a) The set of real numbers under addition is a group.

True False

(b) The set of real numbers under multiplication is a group.

True False

(c) The set of even integers under addition is a group.

True False

(d) The subset of real numbers $\{r \in \mathbb{R} : r \leq -1 \text{ or } r \geq 1 \text{ or } r = 0\}$ under addition is a group.

True False

(e) The subset of real numbers $\{r \in \mathbb{R} : -1 \leq r \leq 1\}$ under addition is a group.

True False

(f) The direct product of any two cyclic groups is cyclic.

True False

(g) The direct product of any two abelian groups is abelian.

True False

(h) If G is not cyclic, then G has a proper subgroup (i.e., a subgroup that's not all of G) that's not cyclic.

True False

(i) If G is cyclic, then all subgroups of G are cyclic.

True False

(j) If G is a finite group of order n and if ℓ evenly divides n, then G has an element of order ℓ.

True False

Section 2: Fill in the blank.

2. (20) *No justification is required for your answers, unless otherwise stated. No partial credit will be awarded.*

(a) Write down the elements of $\mathbb{Z}_{10}$ along with the order of each element. (No justification needed.)

(b) Give an example of a group G and subgroups H and K such that $H \cup K$ is not a subgroup. (No justification needed.)

(c) Give an example of an infinite group G such that every subgroup except $\{e\}$ is also infinite. (No justification needed.)

(d) Give an example of a noncyclic group of order 100 along with a one sentence explanation why your group is noncyclic. (Hint: If G and H are finite, then $|G \times H| = |G| \cdot |H|$.)

(e) Give an example of an infinite nonabelian group G such that $Z(G)$ is finite, but contains more than just the identity. You must state both G and $Z(G)$. Recall that $Z(G)$ means "the center of G". (No justification necessary.)

Section 3: Free Response.

You must show all work to receive credit. If you need more space you may use the back of the page. You must clearly indicate on the front of the page that there is more work on the back of the page. Please work neatly.

3. (15) Let G be a group and let H and K be subgroups of G. Prove that $H \cap K$ is a subgroup of G.

4. (15) Let G be a finite abelian group consisting of the elements $\{a_1, a_2, \ldots, a_n\}$. Define c to be the product of all elements in G: $c = a_1 a_2 \cdots a_n$. Prove that $c^2 = e$, where e is the identity element of G.

5. (15) Let G be a group and let $x, y \in G$. Prove that $o(xy) = o(yx)$.

6. (15)

(a) Let G be a finite group. Suppose that $o(x) = |G|$ for all $x \in G$ except for the identity. Prove that G has no subgroups other than itself and the subgroup containing only the identity.

(b) Prove that a group G satisfying the above criteria must be abelian.