MATH 20 – HOMEWORK 5

due Wednesday, August 2

Instructions: This assignment is due at the *beginning* of class. Staple your work together (do not just fold over the corner). Please write the questions in the correct order. If I cannot read your handwriting, you won't receive full credit.

You may use Wolfram Alpha to compute any necessary sums or integrals.

If you're using facts about distributions to answer the questions, be very clear about which distribution you're using to model that problem and why that distribution is appropriate.

1. When you listen to your "Math Homework" playlist on shuffle on Spotify, you usually hear your favorite song about once every two days. If you then go a whole week without hearing it, how surprised are you? (In other words, what's the probability of this occurring?)

2. On an average 8-hour school day, 1000 people walk into Kemeny Hall. Assume this happens completely randomly[1]. What is the probability that exactly six people enter Kemeny Hall in a ten minute span?

3. Let $X_1, X_2, \ldots, X_k$ be k random variables that are mutually independent and uniformly distributed on the interval $[0, 1]$. Define a new random variable $Y = \min(X_1, X_2, \ldots, X_k)$ such that the value of Y is the smallest of the values of $X_1, X_2, \ldots, X_k$. Find $\mathbb{E}[Y]$.

4. Let X be a discrete random variable that takes only positive integer values. Our normal formula for the expected value of X says
$$\mathbb{E}[X] = \sum_{k=1}^{\infty} kP(X = k).$$
Prove the following alternate formula:
$$\mathbb{E}[X] = \sum_{k=1}^{\infty} P(X \geq k).$$

[1]Of course, this is a terrible assumption—people are more likely to arrive in the short periods between classes. But let's ignore that for now.