

# MATH 20 – HOMEWORK 4

due Wednesday, July 26

**Instructions:** This assignment is due at the *beginning* of class. Staple your work together (do not just fold over the corner). Please write the questions in the correct order. If I cannot read your handwriting, you won't receive full credit.

1. Give an example where Markov's inequality is actually an equality. That is, give a  $\Omega$ ,  $m$ ,  $X$ , and  $a$  such that  $P(X \geq a) = \mathbb{E}[X]/a$ .
2. Prove that for any random variable  $X$ ,

$$\text{Var}(X) = \mathbb{E}[X^2] - (\mathbb{E}[X])^2.$$

3. Suppose  $\Omega = \{a, b\}$ , that  $m(a) = m(b) = 1/2$ , and that  $X$  is a random variable with  $X(a) = a$  and  $X(b) = b$ . Find a formula in terms of  $a$  and  $b$  for  $\mathbb{E}[X]$  and  $\text{Var}(X)$ .
4. Consider flipping a weighted coin with the property that the coin comes up heads with probability  $p$  and tails with probability  $1 - p$ . Suppose that you flip the coin repeatedly until it comes up tails. Let  $X$  be the random variable for the number of flips completed. Find  $\mathbb{E}[X]$  and  $\text{Var}(X)$ . You may use Wolfram Alpha or another tool to compute the value of any infinite sums.
5. Consider flipping a fair coin. Suppose that you flip the coin repeatedly until *you get a heads then a tails consecutively, in that order*. For example, some flipping sequences are: HT, HHHHHT, and TTTTHHHT. Let  $X$  be the random variable for the number of flips completed. Find  $\mathbb{E}[X]$  and  $\text{Var}(X)$ . You may use Wolfram Alpha or another tool to compute the value of any infinite sums.