MATH 20 – HOMEWORK 3

due Wednesday, July 19

Instructions: This assignment is due at the *beginning* of class. Staple your work together (do not just fold over the corner). Please write the questions in the correct order. If I cannot read your handwriting, you won't receive full credit.

1. Suppose you flip a penny and a dime. Let X be the result of flipping the penny where we assign the value of Heads to be 2 and the value of Tails to be 1, and let Y be the result of flipping the dime where we assign the value of Heads to be 4 and Tails to be 3. (So, for example, $X(\text{heads}) = 2$.) Find $\mathbb{E}[X]$, $\mathbb{E}[Y]$, $\mathbb{E}[X+Y]$ and $\mathbb{E}[XY]$. Compute $\text{Var}(X+Y)$ and $\text{Var}(XY)$.

2. Repeat the previous question under the assumption that (through some magical forces), the coins always land on the same side (both heads or both tails). For each answer, compare with the one you found in the previous question: are they bigger, smaller, the same? Explain why this is intuitively true.

3. Suppose you flip a weighted coin that lands on heads with probability p and tails with probability $1-p$. If we let X be the random variable that assign the value of 1 to Heads and the value of 2 to Tails, then what is $\mathbb{E}[X]$? What is $\text{Var}(X)$.

4. Someone offers you the following game: You pay \$1 to play the game. They will shuffle a standard 52-card deck and you will choose a card. If it's a face card, you win \$3 (for a total profit of \$2). Otherwise, you win \$0 (for a total profit of -\$1). Assume for the purposes of this question that the face cards are Jack, Queen, and King, and Ace. Phrase this setup in terms of a sample space Ω and with a random variable X describing your profit. What is $\mathbb{E}[X]$? What is $\text{Var}(X)$? Should you play this game?

5. Someone offers you the following game: You roll a fair six-sided die. If you roll a 1, you win \$25. If you roll a 2, you win \$5. If you roll a 3, you win nothing. If you roll a 4 or a 5, you lose \$10. If you roll a 6, you lose \$15. Should you play this game?