MATH 20 – HOMEWORK 2

due Wednesday, July 12

Instructions: This assignment is due at the *beginning* of class. Staple your work together (do not just fold over the corner). Please write the questions in the correct order. If I cannot read your handwriting, you won't receive full credit.

1. *(4 points)* Two archers, Mary and Paul, are shooting at the same target. Mary hits the target 75% of the time and Paul hits the target 25% of the time. Now suppose that both archers shoot one arrow at the target at the same time. If exactly one arrow hits the target, what is the probability that it was shot by Mary?

2. *(4 points)* Imagine that you have three six-sided dice in a bag. Two are normal, fair, six-sided dice, but the third has the numbers 1, 1, 3, 3, 5, 5 on it. (Still, each of the six sides comes up with equal probability.)

   Now suppose that you randomly pick one of the dice out of the bag, and roll it three times (without peeking to see which of the dice it is). If the three rolls come up two 1s and a 3, in any order, then what is the probability that the die you picked is the one labeled 1, 1, 3, 3, 5, 5?

3. *(4 points)* In order to test whether a batch of 100 widgets meets specifications, a manufacturer picks 20 items at random. If none are defective, the batch is accepted. If at least one is defective, the manufacturer — whose turns out to be quite unscrupulous — mixes the 20 widgets back in with the 100, then picks 20 more at random. If none are defective, the batch is accepted. If at least one is defective, then the manufacturer finally rejects the batch. If a batch of 100 items has 10 defective widgets, what is the probability that it gets accepted?

4. *(4 point)* Sue claims that she can distinguish between Pepsi and Coke 75 percent of the time. Mel bets that she cannot and is just guessing randomly. To settle this a bet is made: Sue is to be given ten small glasses, each having been filled with Pepsi or Coke, chosen by tossing a fair coin. Sue wins the bet if she gets seven or more correct. Find the probability that Mel wins if Sue has the ability that she claims. Find the probability that Mel wins if Sue is guessing.

5. *(4 point)* Determine if the following pairs of events are dependent or independent. (You must prove your answer, not just rely on intuition.)

   (a) $A =$ flipping tails on a fair coin, $B =$ rolling a 3 on a fair six-sided die.
   (b) $A =$ drawing a 7 from a deck of cards, $B =$ drawing a heart from a deck of cards
   (c) $A =$ drawing a 3 from a deck of cards, $B =$ drawing an ace from a deck of cards