

# MATH 20 – HOMEWORK 1

due Wednesday, July 5

**Instructions:** This assignment is due at the *beginning* of class. Staple your work together (do not just fold over the corner). Please write the questions in the correct order. If I cannot read your handwriting, you won't receive full credit.

1. (3 points) In how many different ways can the letters of the word BOOKKEEPER be rearranged? (For example, the letters in the word EYE can be rearranged in three ways: EEY, EYE, and YEE.)
2. (3 points) Suppose that you draw a seven-card hand at random from a standard deck of 52 cards. What is the probability that your hand contains three of one card and four of another (for example: 3, 3, 3, 3, Q, Q, Q or 6, 6, 6, 9, 9, 9, 9)?
3. (3 points) Suppose that  $A$  and  $B$  are events. If  $P(A \cup B) = 5/7$ ,  $P(\bar{B}) = 4/5$ , and  $P(A) = 3/5$ , then what is  $P(A \cap B)$ ?
4. (3 points) Prove that if  $A$  and  $B$  are sets, then

$$\overline{A \cup B} = \bar{A} \cap \bar{B}$$

and

$$\overline{A \cap B} = \bar{A} \cup \bar{B}.$$

*Hint:* One way to prove that two sets  $S$  and  $T$  are equal is to first prove that  $S \subseteq T$  and then prove that  $T \subseteq S$ . The only way both of these are true is if  $S = T$ .

5. (4 points) Give a proof by contrapositive of the following statement.

*If  $x + y$  is even then either  $x$  and  $y$  are both even or  $x$  and  $y$  are both odd.*

6. (4 points) Give a proof by contradiction of the following statement.

*If  $n$  is a positive natural number and  $n^2$  is even, then  $n$  is also even.*