Topic 11 - Simulated Annealing

HW 4 due Mon, 11:59pm

Exploration vs. Exploitation

Diversification vs. Intensification

↓ ↓

Looking in areas of the search space that you haven't seen before

Searching the area you're already in very hard for better and better solutions

Maximally Exploitative: Hill-Climbing

Maximally Explorative: Random Search

Mix: Hill Climbing with Random Restarts

Best MHs combine exploring and exploiting in balanced and clever ways.

MHs = "how to go downhill smartly"

* Simulated Annealing

Hill Climbing: only moves that improve the score are allowed

SA: Some worsening moves are allowed with some probability

* At the start the system has a high temperature.

* The probability of accepting a worse move starts out high.

* Over time, the system cools down, which decreases the prob. of accepting a worsening move.

Very high temp: basically a random walk, accepting all tweaks

Very low temp: basically hill climbing

Idea: as the system cools, you'll eventually wander onto a really good hill and stay there, because the best hills are hardest to leave.

Technical Details:

Acceptance conditions:

Suppose the current temp is T.
Current solution x (assuming maximizing)
Tweak: $s = \text{tweak}(x)$
Define $\Delta = \text{score}(s) - \text{score}(x)$

If $\Delta > 0$, s is an improvement
| always accept
If $\Delta \leq 0$, accept with probability
$p = e^{\Delta/T}$ $e \approx 2.71\ldots$

$\Delta \leq 0$, $T > 0 \Rightarrow \Delta/T \leq 0$
This implies $0 < e^{\Delta/T} \leq 1$

Looks strange but has motivations from physics ("Boltzmann distribution")

What if we're doing a minimization problem? Change things accordingly.

Temperature and Cooling Schedule

The way the temperature changes over time

* Geometric (most common)

  Pick some $0 < \alpha < 1$

  When it's time for the system to cool down, set $T = T \cdot \alpha$

  Ex: Initial temp 10, $\alpha = 0.9$

  $10 \to 9 \to 8.1 \to 7.29 \to \cdots$ (will never hit 0)

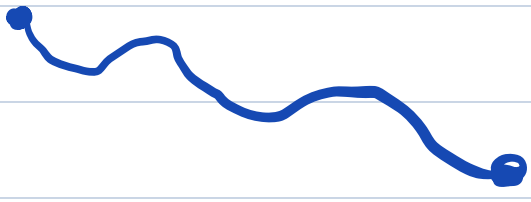
  Initial Temp T_0

  The temperature at time n is $T_n = T_0 \cdot \alpha^n$

* Linear cooling schedule

  $T = T - \beta$ for some β.

* Many more out there including non-monotone



Process:

Pick an initial temperature (How?)

x = random elt. in search space

best = x

Repeat: (How many times?) (How long?)

  For a while:

    s = tweak(x)

    Δ = score(s) − score(x)

    if Δ > 0:

      x = s

      if score(x) > score(best):

        best = x

    else:

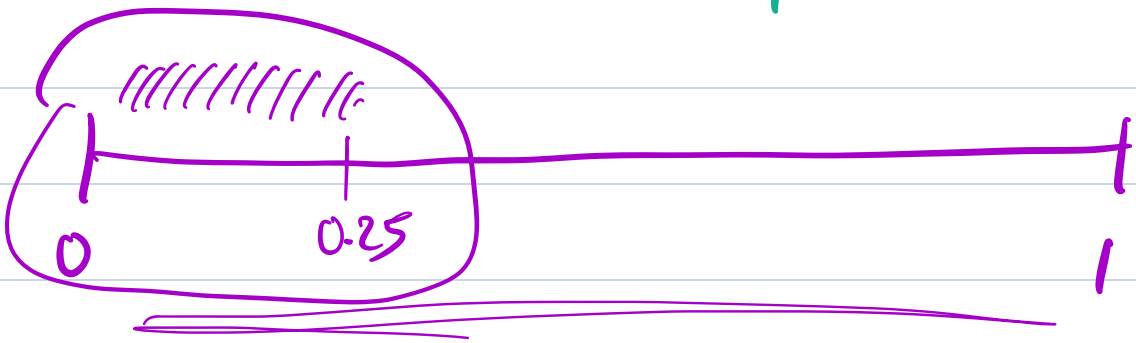
      r = random # in [0,1]

      if r < $e^{\Delta/T}$:

        x = s

  adjust the temperature according to the cooling schedule

If you want to enter an if statement with probability p: 0.25
pick a random # r in [0, 1]
check if r < p



Questions to answer:

* How to pick an initial temp
* How long to loop for each temp
* When to stop
* How to cool

art, not science