

Fri, Apr. 5, 2024
Scientific Computing

①

Announcements

- * Homework 4 due today (or Mon if you have asked for an extension)
- * No in-person lecture on Mon, I will post a video
- No O.H. on Monday.

Topic 11 - Hill-Climbing

TSP with 300 cities

Scoring one element of the search space is not that bad

300 distance calculations

each is two subtractions

two squarings

one addition

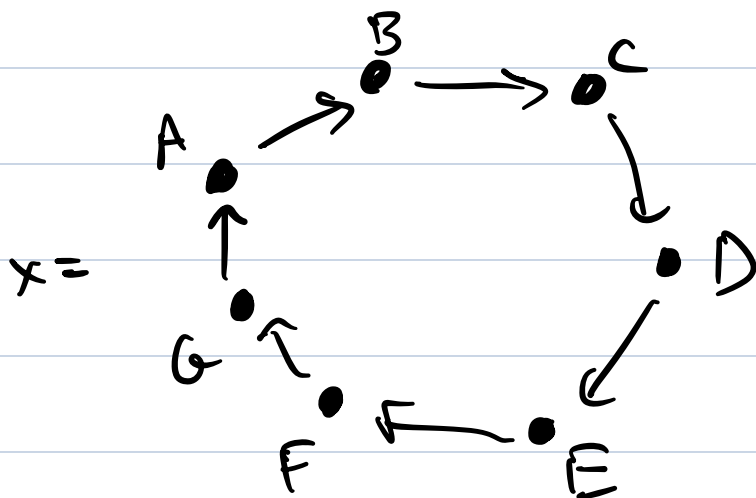
one $\sqrt{\quad}$

Size of nbhd: $\binom{299}{2} = 44,551$

Scoring 44,551 of them is slow.

Demos 3,4

Great trick to speed up scoring.

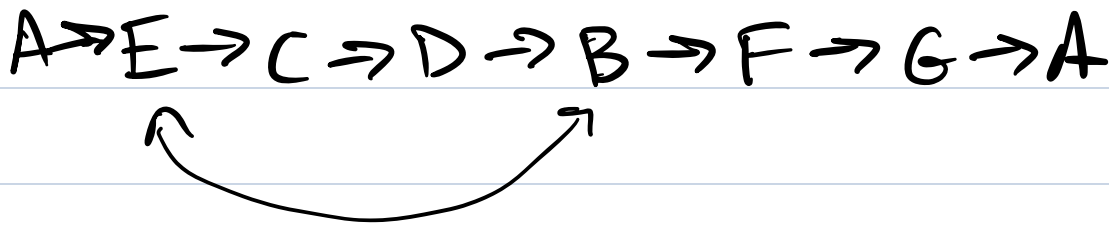
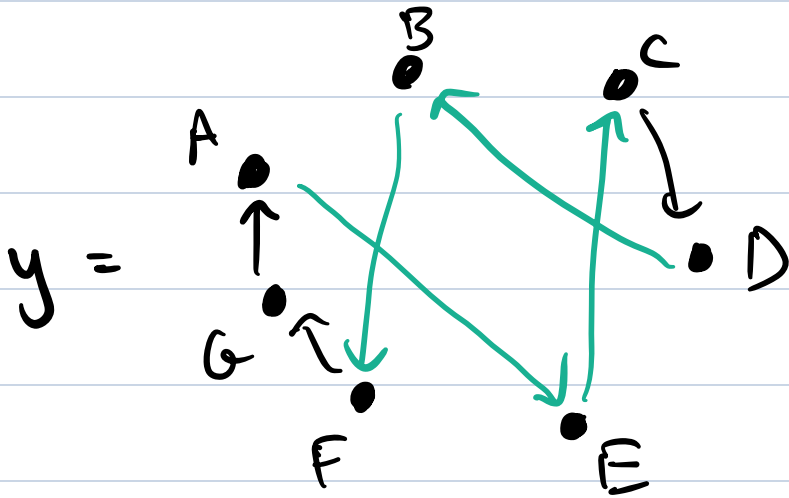


$d = \text{distance function}$

$\text{Score}(x) =$

$$d(A,B) + d(B,C) + d(C,D) + d(D,E) + d(E,F) + d(F,G) + d(G,A)$$

Swap 2 cities, B, E



$$\text{Score}(y) =$$

$$\cancel{d(A,B)} + \cancel{d(B,C)} + d(C,D) + \cancel{d(D,E)} + \cancel{d(E,F)} + d(F,G)$$

$$d(A,E) \quad d(E,C) \quad d(D,B) \quad d(B,F) + d(G,A)$$

If you have 300 cities, still only 4 edges change with this tweak.

new-score = old-score - 4 edges that disappear
+ 4 edges that are new

Old method: 300 distances per score

Now: 8 per score

$$\frac{300}{8} = 37.5x \text{ faster}$$

Next Q: Is it a good tweak function?

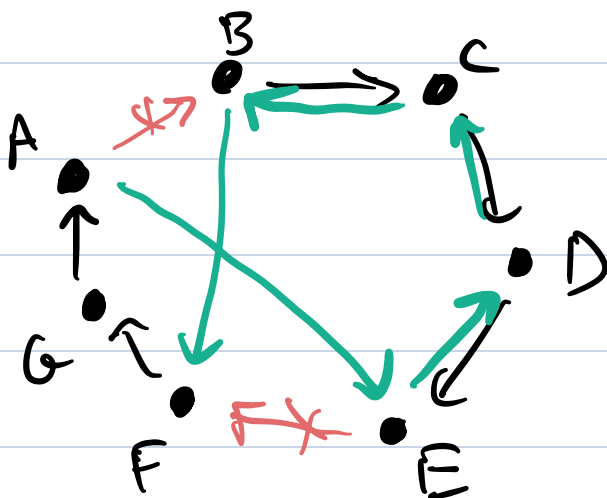
No, it gets stuck on bad solutions.

Moral: smaller tweaks are better

Better: Reverse a block of cities

A → B → C → D → E → F → G → A
reverse

A → E → D → C → B → F → G → A



This changes
2 edges per
tweak, not 4.

Big theme of MHLs: They are super flexible. Any idea you have can be tried.

Can we adapt Steepest Ascent H.C. to continuous spaces?

n-trials Steepest Ascent
MH #3



$x = \text{random elt. of search space}$

while True:

temp = x

repeat n times:

s = tweak(x)

if score(s) > score(temp):

temp = s

x = temp

When $n=1$, just called "Hill Climbing"

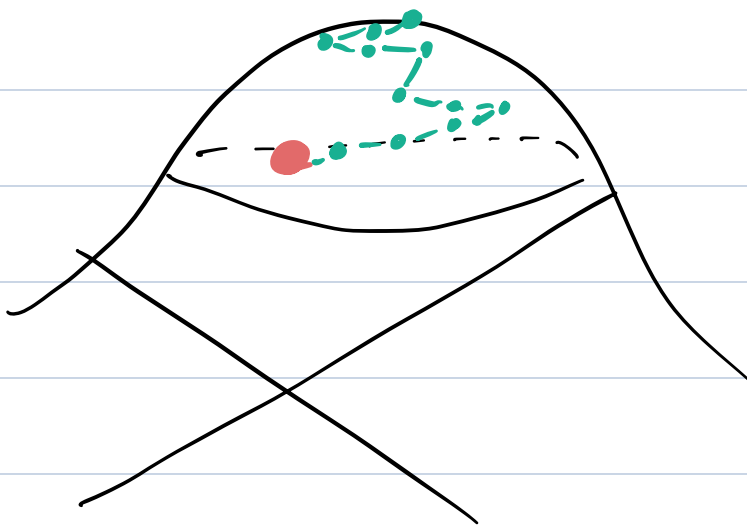
M4 #4 Hill Climbing

$x = \text{random elt. of } S$

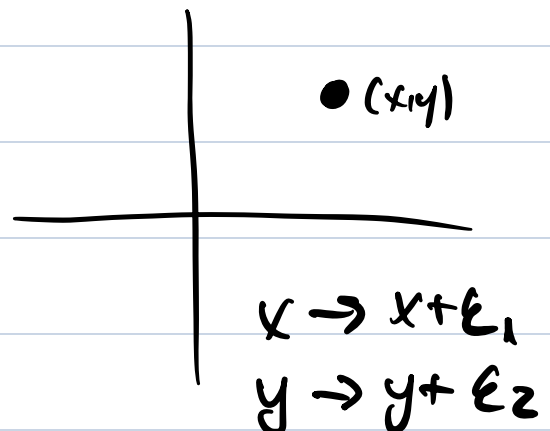
while True:

$s = \text{tweak}(x)$
if $\text{score}(s) > \text{score}(x)$:
 $x = s$

Try a random nearby sol. If better, go there. If not, don't. Repeat.



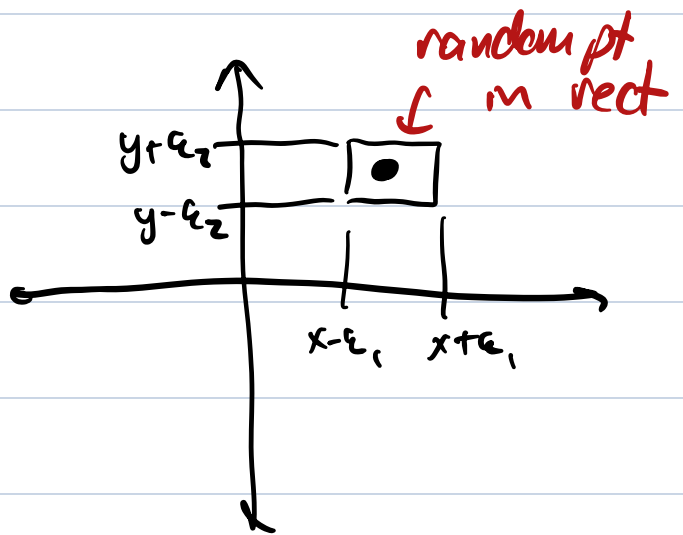
We'll have a lecture about to tweak in continuous space.



ϵ_1 and ϵ_2 are small #s

$$\epsilon_1 \in [-0.001, 0.001]$$

$$\epsilon_2 \in [-0.001, 0.001]$$



TSP scores

	50 cities	vs	300 cities
Steepest Ascent, swap 2	9.87		32.83
S.A., reverse bl.	6.48		14.36
H.C., swap 2	8.42		29.4
H.C., reverse bl.	6.72		14.62

MH #5 Hill-Climbing with random restarts

best = random element of S

while True:

$x = \text{random elt. of } S$
 $s = \text{HillClimb}(x)$

```
if score(s) > score(best):  
    best = s
```