Wednesday, May 3, 2023
Lecture #42
MSSC 6000

Announcements

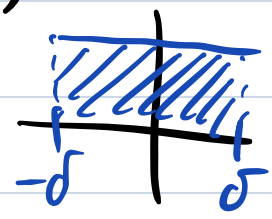
* Homework 6 due the last day of class 11:59pm
* Final Exam (take-home) assigned last day of class, due Friday, May 13, 11:59pm
* Course Evaluations are open
* Normal OH this week
  2:30 - 3:30 today on MS Teams

Topic 15 - Neighborhoods in Continuous Space

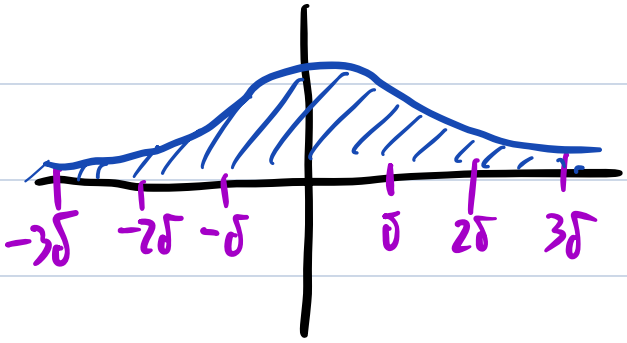
Ways to pick points in continuous space with a d-dimensional sphere of radius 1.

Ways to move around continuous space: ②

1) Pick a random point (uniformly) in a dimensional sphere. $U(-\delta, \delta)$

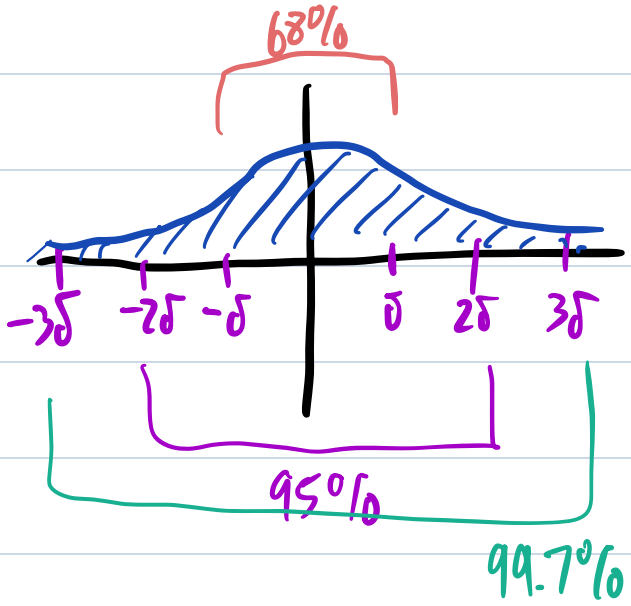


2) Gaussian (normal) random walk:
For each component of $x = (x_1, x_2, \ldots x_d)$ add a shift drawn from a normal distribution with mean 0 and std. dev. δ.



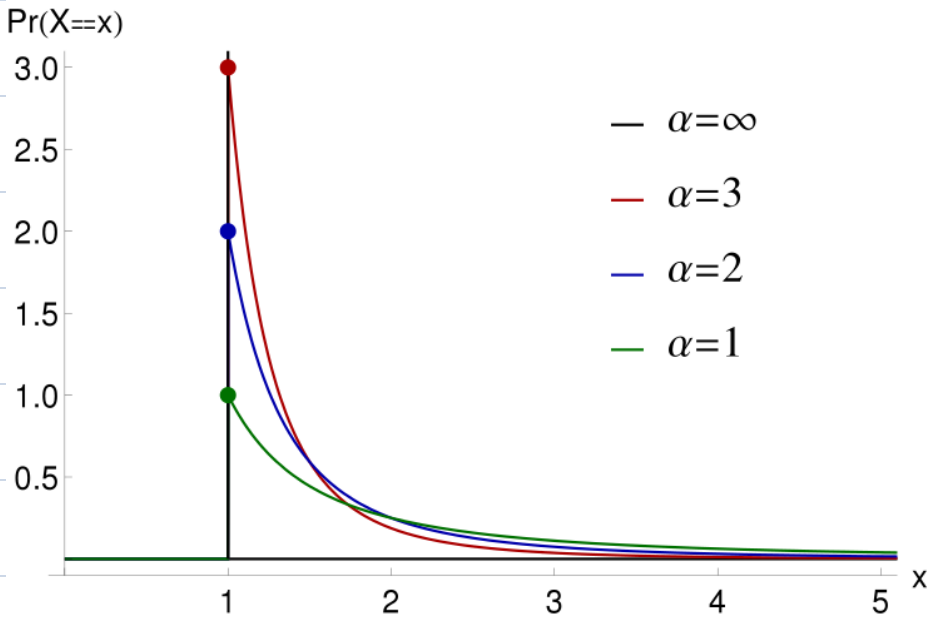
$(x,y) \to (x+s_1, y+s_2)$ where s_1, s_2 are drawn from $N(0,\delta)$.

* smaller tweaks are more likely than larger tweaks
* tweaks can be arbitrarily large (but less and less likely)



"thin tails"

3) Lévy flight



A different distribution.

* Much thicker tails than a normal distribution.

Many many small tweaks but a small probability of an extremely large tweak.

Only gives pos. #s, so we have to also randomly pick a + or − sign to go with it.

To draw a random #: $s = \bar{r}^{1/\alpha}$, where r

is a uniform random # drawn from $[0, 1]$.

When are Levy flights good or bad?

* Bad if α is too small
* Bad if the landscape is very spikey.
* Very bad for H-C
  ↳ useless
* Makes simulated annealing more interesting
  - more exploration
* Great for MHs that have a population of solutions and need a tweak function.

Topic 16 - Firefly Search and Cuckoo Search

Lots of MH are inspired by biology.

The trick is telling which ones are good and which ones are good and which are.... less good.

Fireflies:

Fireflies use their lights to attract each other, and the level of attraction depends on the intensity of the light.

Firefly search is a population MH, kind of like PSO but with different movement rules.

Each firefly represents a solution.
Better solutions = brighter lights

A firefly's movement in each step is toward each other brighter firefly, plus a random component (Gaussian or Lévy flight).

Difference with PSO: We don't compare solutions like this.

Suppose firefly j is brighter than firefly i.
The attractiveness of j to i is:

$$A_{ij} = \beta \cdot e^{-\gamma r_{ij}^2},$$

where r_{ij} is the Euclidean distance between the two fireflies, and β and γ are parameters that we can tune.

Attractiveness decays exponentially according to the square of the distance.

Movement:

In each generation, we loop through all pairs of fireflies and move the dimmer one toward the brighter one:

$$x_i' = x_i + \beta e^{-\gamma r_{ij}^2}(x_j - x_i) + \alpha \cdot s \cdot L$$

- x_i': new pos
- x_i: old pos
- $\beta e^{-\gamma r_{ij}^2}$: A_{ij} attractiveness
- $(x_j - x_i)$: vector from x_i to x_j
- α: tuning param
- s: random vector of ± 1
- L: levy flight in d-dimensions

Pseudocode:

```
N = # of fireflies
while True:
    for i from 1 to N:
        for j from 1 to N:
            if firefly j is brighter than i:
                move i toward j according
                to the formula
```