Monday, May 1, 2023
Lecture #41
MSSC 6000

Announcements

* Homework 6 due the last day of class 11:59pm
* Final Exam (take-home) assigned last day of class, due Friday, May 13, 11:59pm
* Course Evaluations are open
* Normal OH this week
  1pm-2pm today in person.

Topic 15 — Neighborhoods in Continuous Space

In our MHs in continuous space that need a tweak function, we've used a simple one:

Start with $x = (x_1, x_2, \ldots x_d)$

(in d-dimensional space)

$$s = \text{tweak}(x)$$
$$= x + (r_1\delta_1, r_2\delta_2, \ldots, r_d\delta_d)$$

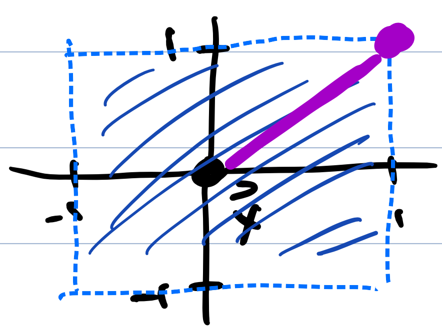
where each r_i is a uniform random # between -1 and 1 and δ_i is a predetermined scaling factor that determines the max. change allowed.

* In most of our examples the x and y bounds were the same, so we used $\delta_1 = \delta_2 = 0.1$ or $\delta_1 = \delta_2 = 0.01$, etc.

* In the spring problem: $\delta_1 = \delta_2 = 0.01$
$\delta_3 = 0.1$

From now on, assume $\delta_i = 1$ and $x = \vec{0}$
$\Rightarrow$ $s = \text{tweak}(x) = (r_1, r_2, \ldots, r_d)$.

2-d space

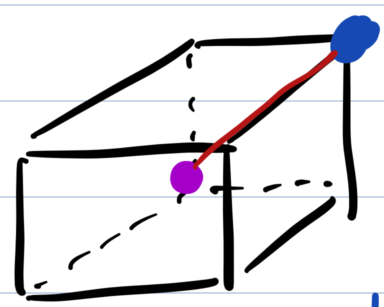


In 2d this tweak randomly picks a pt. in a square with side length 2.
(uniformly)

What is the furthest that s could be from x (in the 2d case)

In the corner, which has distance $\sqrt{2}$.

In 3D:



$s = (1,1,1)$

$x = (0,0,0)$

distance $\sqrt{3}$

In Dimension d: the furthest away that s can be from x is $\sqrt{d}$

As dimension grows, the tweaks can be further and further away, which isn't good.

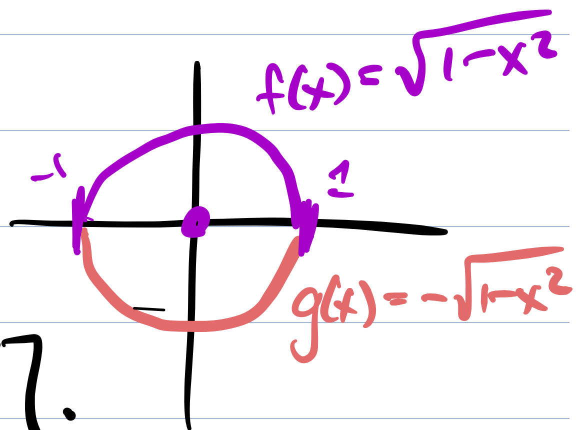
One way to fix this:

* scale s down by a factor of $\frac{1}{\sqrt{d}}$.

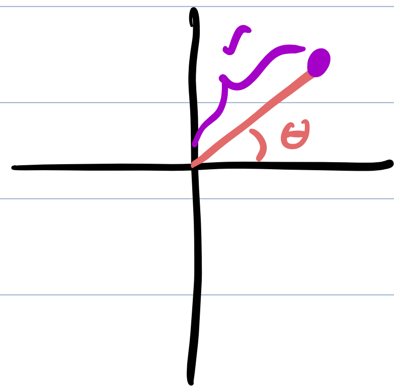
Scales everything equall

* Instead of picking a random point in a square, pick a random point in a circle.

You have to be very careful about how you randomly pick points in a circle.



Ex: Pick $x \in [-1,1]$ uniformly then pick $y \in [-\sqrt{1-x^2}, \sqrt{1-x^2}]$.
(bad, demo)



Ex: Pick $\theta \in [0, 2\pi)$, then pick $r \in [0,1]$.

Ex: Rejection Sampling
* Pick a point uniformly in the square.
* If it's in the circle, keep it, otherwise

try again.

How inefficient is this?

Area of the 2D square? 4

Area of the 2D circle? $\pi \approx 3.14$

The probability that a point in the square is also in the circle is $\frac{\pi}{4} \approx 75\%$.

The volume of the d-dim. cube vs. the d-dim. square:

d	vol. of d sphere	vol. of d-cube	
2	3.14	4	75%
3	4.19	8	50%
4	4.93	16	25%
5	5.26	32	
6	5.168	64	
10	2.55	1024	
20	0.03	1,048,576	

rejection sampling can't work in higher dimensions.

"Muller method" Mullet method

Weird but it works:
To pick points uniformly from a d-dimensional sphere:

- pick $(u_1, u_2, \ldots, u_d)$
  each coordinate from a Gaussian (normal) distribution with mean 0 and std. dev. of 1
- Define $\text{norm} = \sqrt{u_1^2 + u_2^2 + \cdots + u_d^2}$
- Define $r = [\text{uniform random } \# \text{ in } [0,1]]^{1/d}$
- Define $\underbrace{x = \dfrac{r \cdot u}{\text{norm}}}_{\text{random point}}$.