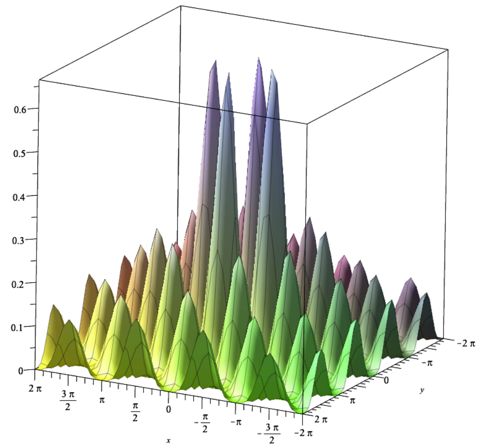
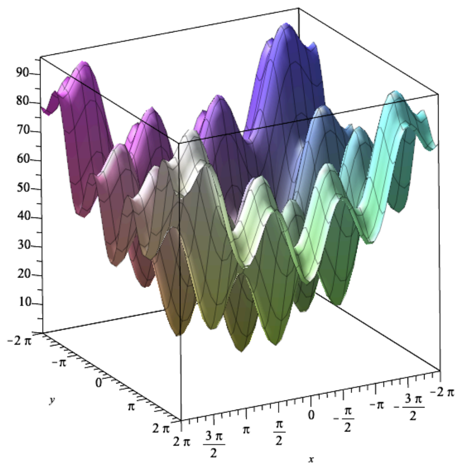
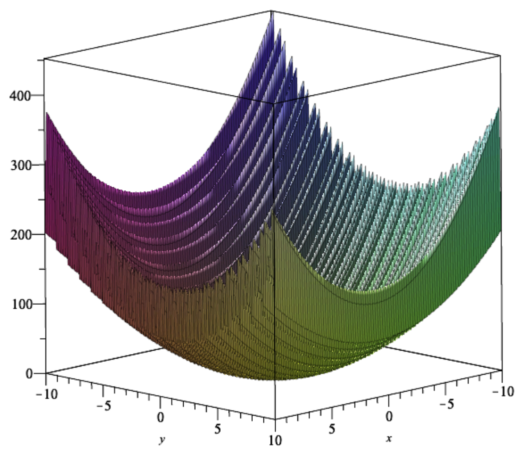


$$\frac{\sin^2(x-y)\sin^2(x+y)}{\sqrt{x^2+y^2}}$$

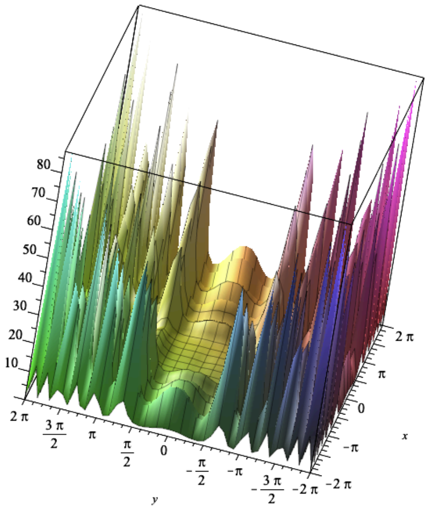


$$x^2+y^2+25\left(\sin^2(x)+\sin^2(y)\right)$$

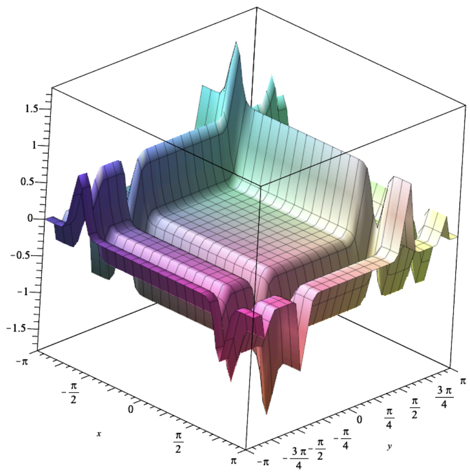


$$\sin^2(3\pi x)+(x-1)^2\left(1+\sin^2(3\pi y)\right)$$

$$+\,(y-1)^2\left(1+\sin^2(2\pi y)\right)$$



$$(|x|+|y|)\,e^{(-\sin^2(x)-\sin^2(y))}$$



$$\sin(y)\sin^{20}\left(\frac{2y^2}{\pi}\right)$$
$$-\sin(x)\sin^{20}\left(\frac{x^2}{\pi}\right)$$

Friday, April 21, 2023
Lecture #37
MSSC 6000

Announcements

* Homework 5 in progress, due Mon 11:59pm
* Homework 6 assigned Monday, due last day of class.
* Final Exam (take-home) assigned last day of class, due TBD ($\approx$ Friday, May 13, 11:59pm)

Topic 13 — Particle Swarm Optimization (PSO)

In H-C and simulated annealing (and many other MHs), we have a tracked a single solution moving through the search space.

PSO (1995) is our first example

of a "population metaheuristic" —
we will track many solutions at a time and they will interact with each other.

Idea: You have N particles, each representing a possible solution, and they start at random positions.

Each particle has a velocity which depends on three things:

1) its current velocity (inertia)
2) the best solution that that particle has ever seen
3) the best solution that any particle has ever seen

Let $x_i(t)$ and $v_i(t)$ denote the position (x) and velocity (v) of particle i at time t.

$$x_i(t+1) = x_i(t) + v_i(t+1)$$

(the velocity determines how the position changes)

$$v_i(t+1) = \alpha \cdot v_i(t) + \beta \cdot r_1 \cdot (b_i(t) - x_i(t)) + \gamma \cdot r_2 \cdot (B(t) - x_i(t))$$

$b_i(t)$ = best solution particle i has seen by time t

$B(t)$ = best solution any particle has seen by time t.

α: weighting factor for inertia ≈ 0.9

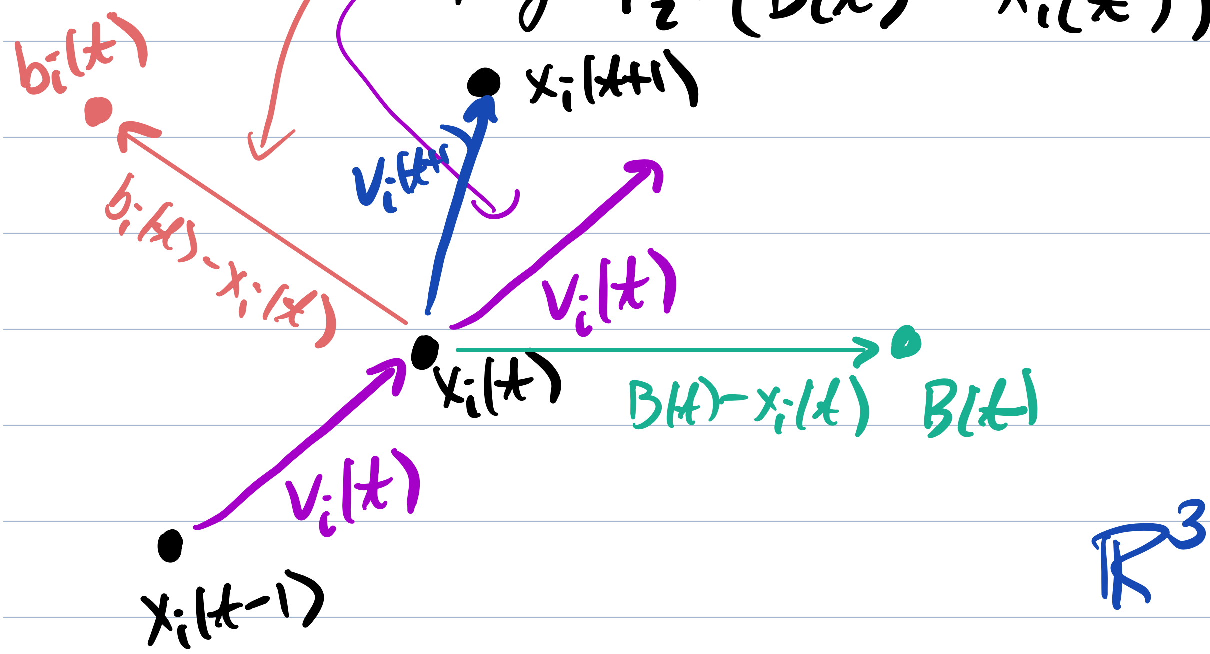
β: weighting factor for personal best ≈ 1

γ: weighting factor for global best ≈ 1

$b_i(t) - x_i(t)$: the vector that points from $x_i(t)$ (current location) to $b_i(t)$ (personal best)

$B(t) - x_i(t)$: the vector that points from $x_i(t)$ to $B(t)$

$$v_i(t+1) = \alpha \cdot v_i(t) + \beta \cdot r_1 \cdot (b_i(t) - x_i(t)) + \gamma \cdot r_2 \cdot (B(t) - x_i(t)) \quad (4)$$



r_1 and r_2 are random vectors in $[0,1]$ $[0.9, 1]$

$$r_1 = \begin{bmatrix} 0.1279\ldots \\ 0.5263\ldots \\ 0.4141\ldots \end{bmatrix} \qquad r_2 = \quad \ldots$$

This makes a lot of sense for continuous problems, no sense for discrete problems.

Vector arithmetic: $b_i(t) = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$ $\quad x_i(t) = \begin{bmatrix} d \\ e \\ f \end{bmatrix}$

$$b_i(t) - x_i(t) = \begin{bmatrix} a-d \\ b-e \\ c-f \end{bmatrix}$$ (5)

$$r_1 = \begin{bmatrix} s_1 \\ s_2 \\ s_3 \end{bmatrix}$$

$$r_1 \cdot (b_i(t) - x_i(t))$$

$$= \begin{bmatrix} s_1(a-d) \\ s_2(b-e) \\ s_3(c-f) \end{bmatrix}$$

$$\beta \cdot r_1 \cdot (b_i(t) - x_i(t))$$

$$= \begin{bmatrix} \beta \cdot s_1 \cdot (a-d) \\ \beta \cdot s_2 \cdot (b-e) \\ \beta \cdot s_3 \cdot (c-f) \end{bmatrix}$$

* Demos