Wednesday, April 12, 2023
Lecture #33
MSSC 6000



## Announcements

* Normal Office Hours today, 2:30-3:30 (Teams)
* Homework 5 assigned, two Simulated Annealing problems — be creative! Bonus pts!

## Topic 12 — Simulated Annealing (continued)

Last time:
- Springs Demo
- Knapsack Demo

There are lots of research papers using SA in interesting applications - airline routing, school bus scheduling, etc.

various ways to handle tweaks that 
violate constraints
* retweak until good
* allow violations, but penalize

The idea of allowing (but penalizing) solutions that violate the constraints allows us to apply SA to problems that have no score, only constraints.

Ex: Solving Sudokus

Score = (# of row conflicts) + (# of col. conflicts) + (# of 3x3 square conflicts)

Goal: Minimize the score

If we get a score of 0, that's a solution.
If we end up with a score $> 0$, we failed.

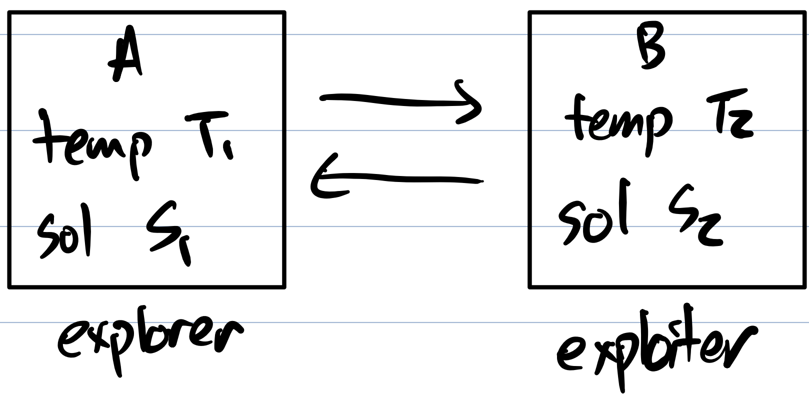
# Parallel Tempering

A variation on SA:

Instead of running one system that cools over time, run multiple systems that are each at a constant temp (but diff. from each other), that are allowed to swap solutions.

**Intuition:** Person A very good at exploring.
Person B very good at exploiting.

They both run for a while, until person A says "I think I found a good hill, let's swap so you can exploit it."




$T_1 > T_2$

Should they swap? Let $E_i = \text{score}(S_i)$

At any point in time, swap with prob

$$p = \min(1, e^{\Delta})$$

where

$$\Delta = (E_1 - E_2) \cdot \left(\frac{1}{T_2} - \frac{1}{T_1}\right)$$

$\frac{1}{T_2} > \frac{1}{T_1}$

$(E_1 - E_2)$: >0 when the explorer has a better solution than the exploiter

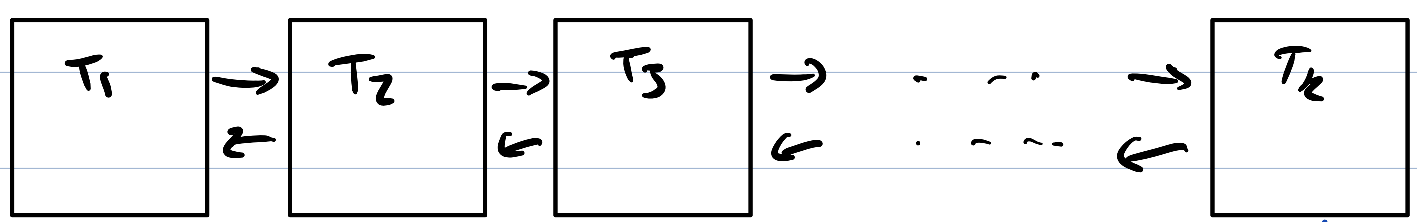
$\left(\frac{1}{T_2} - \frac{1}{T_1}\right)$: always >0

In this case, $p=1$, always swap

If $E_1 > E_2$, always swap.
If $E_1 < E_2$, sometimes swap depending on how much worse $E_1$ is.

## More generally, k different systems:

hottest — coldest




most explorative — most exploitative

System $i$ and $i+1$ swap with prob

$$p_i = \min(1, e^{\Delta_i})$$

$$\Delta_i = (E_i - E_{i+1}) \cdot \left( \frac{1}{T_{i+1}} - \frac{1}{T_i} \right)$$

* Can also allow the temps to vary if swaps are happening too often ($> 2\%$) or too rarely ($< 0.1\%$).