Fri, March 31, 2023
Lecture #29
MSSC 6000

Announcements

* HW 4 due Mon, Apr 3, 11:59pm
* Fri, Apr. 7 - no class
  Mon, Apr. 10 - no lecture (home work day)
  no OH

Topic 12 - Simulated Annealing

Spoons

Hill Climbing: only allows moves that improve the score (sometimes with random restarts)

SA: worsening moves are accepted with some probability

At the start, the "system" has a "high temperature" and the probability of accepting a worse move is high.

Over time the system "cools" and the probability decreases slowly to 0.

Very high temp: basically doing a random walk, accepting every tweak

Very low: basically hill climbing

Idea: as the system cools you hope to wander onto a good hill and get <u>stuck</u> there.

Technical Details

Acceptance conditions

(assume maximizing)

Suppose the current temp is T.

x = current sol

s = tweak(x) (maybe the new sol)

Define Δ = score(s) − score(x)

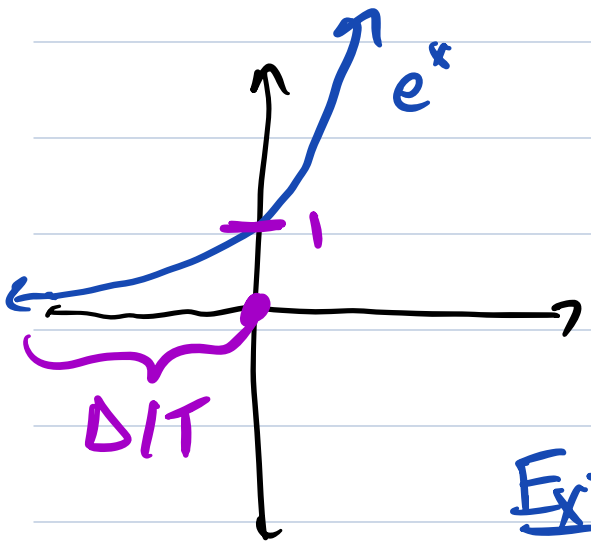
If $\Delta > 0$, s is an improvement over x, always accept

If $\Delta \leq 0$, accept with probability $p = e^{\Delta/T}$

↳ from physics

Since $\Delta \leq 0$, $\Delta/T \leq 0$

$\Rightarrow 0 < e^{\Delta/T} \leq 1$



T being higher $\Rightarrow \frac{\Delta}{T}$ is lower

Ex: $\Delta = -5$ $T = 1 \Rightarrow \frac{\Delta}{T} = -5$

$\Delta = -5$ $T = \frac{1}{10} \Rightarrow \frac{\Delta}{T} = -50$

Cooling Schedule:

The way the temperature changes over time.

* We control this!

* Geometric (most common)

Pick some # $0 < \alpha < 1$ ahead of time.

new temp = (old temp) $\cdot \alpha$

Ex: initial temp = 10 $\alpha = 0.9$

$10 \to 9 \to 8.1 \to 7.29 \to \cdots$

(will never hit 0)

$\alpha = 0.9$ is pretty fast

$\alpha = 0.95, 0.98, 0.99$ is good

$$T_n = T_0 \cdot \alpha^n$$

T_n: temp after n coolings; T_0: initial temp

Linear: Pick a number $\beta > 0$

new temp = (old temp) $- \beta$

If you cool too much, you'll have negative temperatures.

↳ suddenly you accept every worse solution

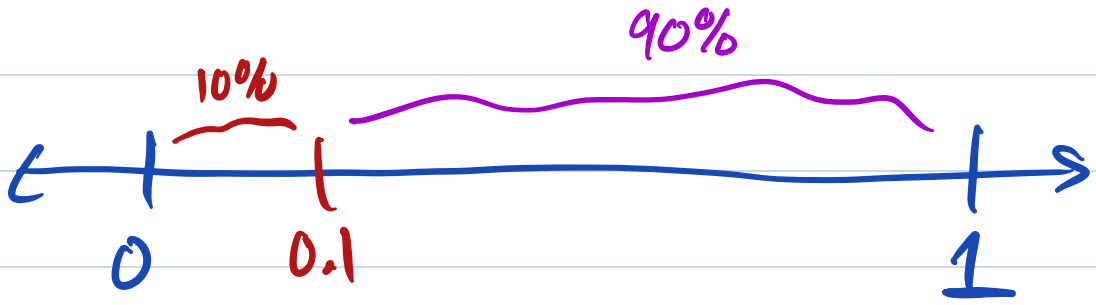
* Many more cooling schedules, including

non-monotone ones.

Process:

Pick an initial temperature T. (How?)

```
x = random solution
best = x
Repeat:                                   (How long?)
    For a while:                          (How long?)
        s = tweak(x)
        Δ = score(s) - score(x)
        if Δ > 0:
            x = s
            if score(x) > score(best):
                best = x
        else:
            r = random # in [0, 1]
            if r < e^(Δ/T):        0.1
                x = s
    adjust the temp according to the
        cooling schedule
```



with probability 10% I want to print "hi"

```
if random() < 0.1:
    print("hi")
```