

MATH 4931 / 5931 – SPECIAL TOPICS: THEORY OF COMPUTATION AND FORMAL LANGUAGES

HOMEWORK 5

Spring 2023

due Friday, May 5, by the beginning of class

This homework assignment was written in L^AT_EX. You can find the source code on the course website.

You must explain your reasoning for all of your answers. Correct answers with no justification or explanation will not be accepted.

Students enrolled in Math 4931: Complete any 4 out of the 7 problems.

Students enrolled in MSSC 5931: Complete any 5 out of the 7 problems.

1. Let $N_\ell(w)$ denote the number of occurrences of the letter ℓ in the word w . Consider the language

$$\mathcal{L}_1 = \{w\#0^{N_0(w)} : w \in \{0,1\}^*\}$$

over the alphabet $\Sigma = \{0,1,\#\}$. These are words that have a word w over $\{0,1\}$, then a $\#$, then some 0's, and the number of 0's after $\#$ must match the number of 0's in w .

Prove that \mathcal{L}_1 is a context-free language by finding a CFG for it.

2. Let $N_\ell(w)$ denote the number of occurrences of the letter ℓ in the word w . Consider the language

$$\mathcal{L}_2 = \{x\#y : x, y \in \{0,1\}^* \text{ and } N_0(x) = N_1(y)\}.$$

Prove that \mathcal{L}_2 is a context-free language by finding a CFG for it.

3. Convert the CFG below into a CFG in Chomsky Normal Form that generates the same language:

$$\begin{aligned} A &\rightarrow BAB \mid B \mid \varepsilon \\ B &\rightarrow 00 \mid \varepsilon \end{aligned}$$

4. Find a pushdown automata for the language \mathcal{L}_1 in Exercise 1. Draw its diagram and also give its formal definition.
5. Find a pushdown automata for the language \mathcal{L}_2 in Exercise 2. Draw its diagram and also give its formal definition.
6. Use the Myhill-Nerode Lemma (not the Pumping Lemma) to prove that the language

$$\{ww : w \in \{a,b\}^*\}$$

is not regular.

7. Use Moore's Algorithm to find the minimal DFA that accepts the same language as the DFA below.

