MATH 4931 / 5931 – SPECIAL TOPICS: THEORY OF COMPUTATION AND FORMAL LANGUAGES

HOMEWORK 3

Spring 2023

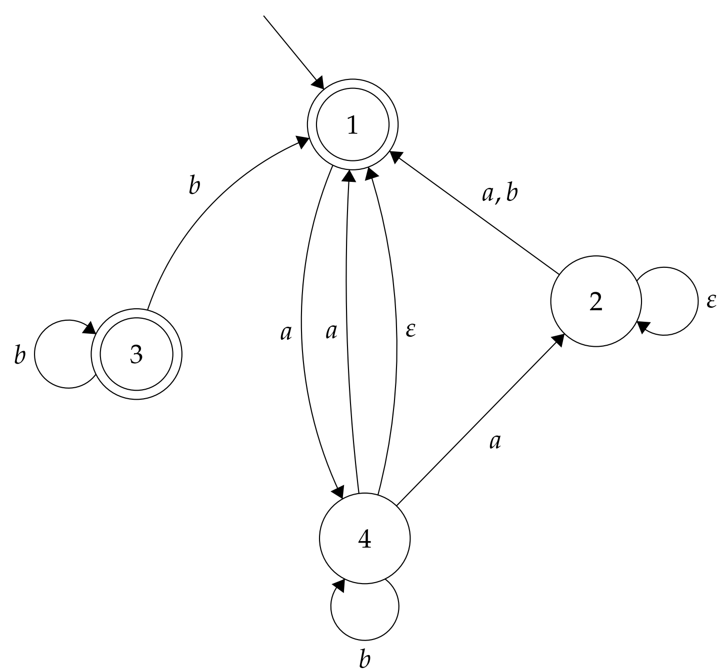
due Friday, March 10, by the beginning of class

This homework assignment was written in LaTeX. You can find the source code on the course website.

You must explain your reasoning for all of your answers. Correct answers with no justification or explanation will not be accepted.

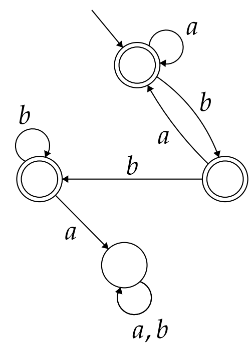
Students enrolled in Math 4931: Complete any 6 out of the 9 problems.
Students enrolled in MSSC 5931: Complete any 7 out of the 9 problems.

1. Convert the NFA below into a DFA that accepts the same language. Remove any unreachable states, but do not simplify further.



2. Use the construction of Lemma 1.55 to find an NFA that accepts the same language as the regular expression $(\{a\} \cup \{b\})^*(\{aa\} \cup \{bb\})(\{a\} \cup \{b\})^*$. Don't attempt to simplify. Your answer will have many states—draw it on a separate paper first so that you can draw it very neatly on your final submission.

3. Use the construction of Lemma 1.60 to find a regular expression that accepts the same language as the DFA below. *Show the GNFAs in each intermediate step.*



4. For any language $\mathcal{L}$, define the language $\texttt{Length}(\mathcal{L})$ over the single-letter alphabet $\Sigma = \{\#\}$ by:

$$\texttt{Length}(\mathcal{L}) = \{w \in \Sigma^* : |w| = |v| \text{ for some } v \in \mathcal{L}\}.$$

Prove that if $\mathcal{L}$ is regular then so is $\texttt{Length}(\mathcal{L})$.

5. Let $\mathcal{L}$ be a language of words over an alphabet Σ and assume $1 \in \Sigma$. Define $\texttt{insert1}(\mathcal{L})$ to be the language of words you get by inserting a single 1 into every word of $\mathcal{L}$ in every possible way. Formally,

$$\texttt{insert1}(\mathcal{L}) = \{x \circ 1 \circ y : x \circ y \in \mathcal{L}\}.$$

Prove that if $\mathcal{L}$ is regular, then so is $\texttt{insert1}(\mathcal{L})$. (Note: we did a simpler version of this in class in which we assumed $1 \notin \Sigma$.)

6. For two languages $\mathcal{F}$ and $\mathcal{G}$, define $\texttt{MixedShuffle}(\mathcal{F})$ to be the language of words that you get by mixing up words from $\mathcal{F}$ and $\mathcal{G}$ back and forth. (This is like the "perfect shuffle" language from Homework 2, except that one interleaved two words by alternating letters back and forth, this one interleaves two words using chunks of letters at a time.) Formally,

$$\texttt{MixedShuffle}(\mathcal{F},\mathcal{G}) = \{w : w = f_1 \circ g_1 \circ f_2 \circ g_2 \circ \cdots \circ f_k \circ g_k \text{ where } f_1 \circ \cdots \circ f_k \in \mathcal{F} \text{ and } g_1 \circ \cdots \circ g_k \in \mathcal{G}\}.$$

For example if $abb \in \mathcal{F}$ and $acdc \in \mathcal{G}$, then one example of a word in $\texttt{MixedShuffle}(\mathcal{F},\mathcal{G})$ is $ababcdc$ because

$$ababcdc = ab \circ a \circ b \circ cdc,$$

which has the form $f_1 \circ g_1 \circ f_2 \circ g_2$ where $f_1 \circ f_2 = abb \in \mathcal{F}$ and $g_1 \circ g_2 = acdc \in \mathcal{G}$.

Prove that $\mathcal{F}$ and $\mathcal{G}$ are regular, then so is $\texttt{MixedShuffle}(\mathcal{F},\mathcal{G})$.

7. Prove that if $\mathcal{L}$ is regular then so is $\{w : w \circ w^R \in \mathcal{L}\}$. (Recall that w^R means the reverse of w.)

8. Consider the language

$$\mathcal{S} = \{x \circ y : x \in \{a,b\}^* \text{ and } y \in \{a,b\}^* \text{ and the number of } a\text{'s in } x \text{ is equal to the number of } b\text{'s in } y\}.$$

Prove that $\mathcal{S}$ is regular.

9. Assume $\Sigma = \{0,1\}$. Let

$$\mathcal{L}_1 = \{w : w \text{ begins with a 1 and ends with a 0}\}$$

and let

$$\mathcal{L}_2 = \{w : w \text{ contains at least three 1s}\}.$$

   (a) Find an NFA for $\mathcal{L}_1 \cup \mathcal{L}_2$.
   (b) Find an NFA for $\mathcal{L}_1 \circ \mathcal{L}_2$.
   (c) Find an NFA for $(\mathcal{L}_2)^*$.