MATH 4931 / 5931 – SPECIAL TOPICS: THEORY OF COMPUTATION AND FORMAL LANGUAGES

HOMEWORK 1

Spring 2023

due Friday, February 10, by the beginning of class

This homework assignment was written in LaTeX. You can find the source code on the course website.

You must explain your reasoning for all of your answers. Correct answers with no justification or explanation will not be accepted.

Students enrolled in Math 4931: Complete any 6 out of the 7 problems.
Students enrolled in MSSC 5931: Complete all 7 problems.

1. Find a regular expression for the language $\mathcal{L}_1$ of words over the alphabet $\{a, b\}$ that have no consecutive aa.
2. Find a regular expression for the language $\mathcal{L}_2$ of words over the alphabet $\{x, y, z\}$ that start and end with the same letter.
3. Find a regular expression for the language $\mathcal{L}_3$ of words over the alphabet $\{0, 1\}$ in which the letters alternate between 0 and 1—in other words, there are no two 0s in a row and no two 1s in a row.
4. Make up a language of words that you think might be regular, and then find a regular expression for it. Be creative!
5. The *reverse* of a word $w = w_1 w_2 \cdots w_n$, denoted w^R is the word $w_n \cdots w_2 w_1$ (i.e., just reverse the order of the letters in the word). If $\mathcal{L}$ is a language, define the *reverse* of $\mathcal{L}$, denotes $\mathcal{L}^R$ to be the set of reverses of words in $\mathcal{L}$:
$$\mathcal{L}^R = \{w^R : w \in \mathcal{L}\}.$$
Prove that if $\mathcal{L}$ is regular, then so is $\mathcal{L}^R$.
6. A regular expression for words over $\Sigma = \{a, b\}$ that contain the pattern aab consecutively is
$$\Sigma^* aab \Sigma^*.$$
Find a regular expression for words over Σ that *don't* contain aab consecutively.
7. For each of the pairs below, decide whether the two regular expressions represent the same language, or different languages. As always, explain your reasoning!

(a)	$(0 \cup 1)^*$	$0^* \cup 1^*$
(b)	$0(120)^*12$	$01(201)^*2$
(c)	$\emptyset^*$	ε^*
(d)	$(0^*1^*)^*$	$(0^*1)^*$
(e)	$\{01, 0\}^*0$	$0\{10, 0\}^*$