Topic 15 - Points in Continuous Space (Continued)

Monday, April 25

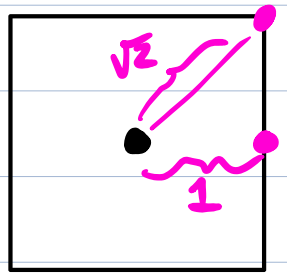
Announcements:

- → HW 5 assigned, due the last day of class
- → Final will be takehome, due Mon, May 16, 11:59pm

For now, assume $x = \vec{0}$, and $\delta_i = 1$ for all i.

$s \leftarrow \text{tweak}(t) = (r_1, r_2, \ldots, r_d)$

The new point s is somewhere in the d-dimensional cube with side length 2 centered at the origin.



What is the distance from the center of a d-dim. cube to a corner?

$(0,0,0,\ldots,0) \rightarrow (1,1,1,\ldots,1)$

$\sqrt{d}$

20-dimensional problem $\sqrt{20}$

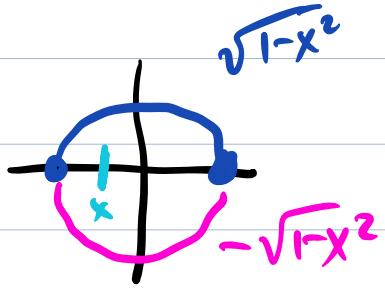
Alternatives:

* Scale down by a factor of $\frac{1}{\sqrt{d}}$.
* Instead of a square, move to a random point inside a circle of radius 1.

How?

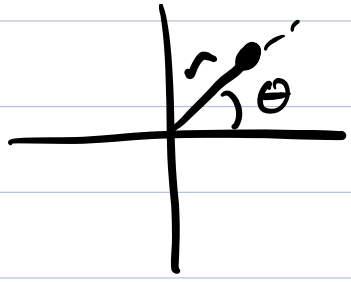
Want a random point in a circle (uniformly)

Ex 1: Pick $x \in [-1, 1]$ uniformly.
Pick $y \in [-\sqrt{1-x^2}, \sqrt{1-x^2}]$ uniformly



bad

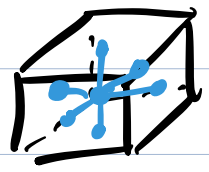
Ex 2: $\theta \in [0, 2\pi)$ uniformly
$r \in [0, 1]$ uniformly



Ex 3: Rejection Sampling
Pick a point uniformly in the

square, if it's in the circle, return it. Otherwise try again.

d	vol of the d-sphere	vol of d-cube
1	2	2
2	3.14	4
3	4.19	8
4	4.93	16
5	5.26	32
6	5.168	64
10	2.55	1024
20	0.03	1,048,576



Solution: How to sample from a d-dim sphere uniformly: "Muller method"

- pick $(u_1, u_2, \ldots, u_d)$ each from the Gaussian (normal) distribution with mean 0 and std. dev. 1.
- set norm $= \sqrt{u_1^2 + u_2^2 + \cdots + u_d^2}$
- set $r = [\text{random in } [0,1]]^{1/d}$
- $x = r \cdot u / \text{norm}$.

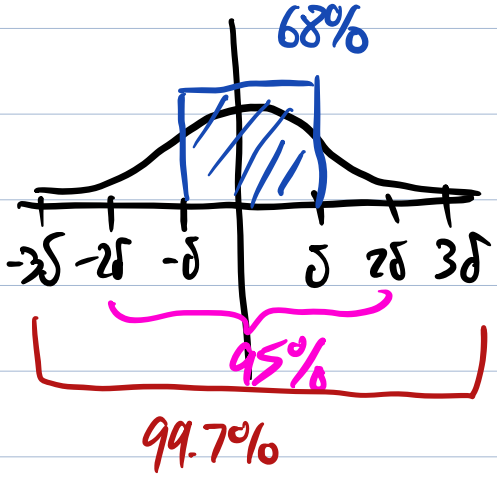
Other ways to move around space:

1) Gaussian Random Walk

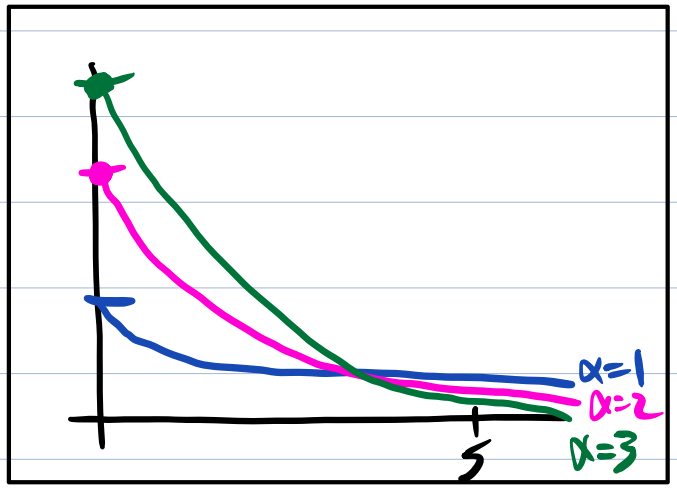
For each component, add a shift drawn from a normal dist. with mean 0 and std. dev. δ

Avg. shift $< \delta$

But small poss. of larger shifts.



2) Lévy flight



A different distr. with much thicker tails ⇒ higher probs. of really big jumps.

Only gives positive #s, so have to randomly pick a +/− sign.

To sample:

$s = r^{1/\alpha}$, where r is uniformly drawn from $(0,1]$, and α is your parameter.

pick + or - sign with equal probability.

Levy flights model how predaters search for prey.

Topic 16 - Firefly Search and Cuckoo Search

Firefly Search

Fireflies use their lights to attract each other, and the level of attraction depends on the intensity of the light.

Population Metaheuristic, similar to PSO with different movement rules.

Each firefly represents a solution (a point in space). A firefly's movement in each step is toward every other brighter firefly (brighter = better solution), plus a

random component (Lévy flight or Gaussian walk).