Friday, April 1 — Day 27

Continuing Topic 11 - Hill Climbing

→ Next week: video lectures, will email
→ HW 4: 1 Q assigned Monday on B+B,
      will be due Wed, Apr 13, 11:59pm
→ No normal OH next week, may schedule
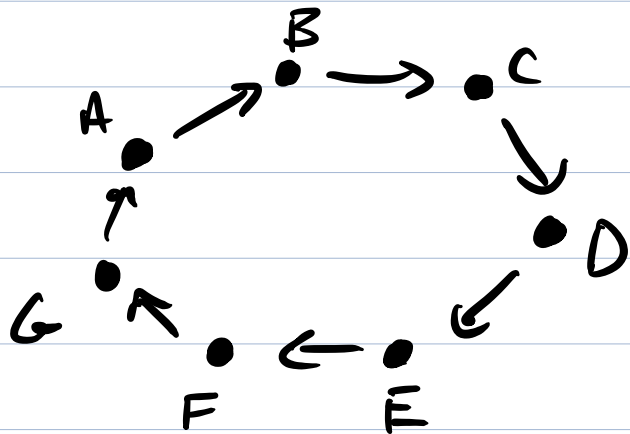   on Tuesday morning, maybe Thurs. mornin

Scoring = slow part

To speed up scoring, think about what your tweak function actually changes.

Suppose we have a tour:
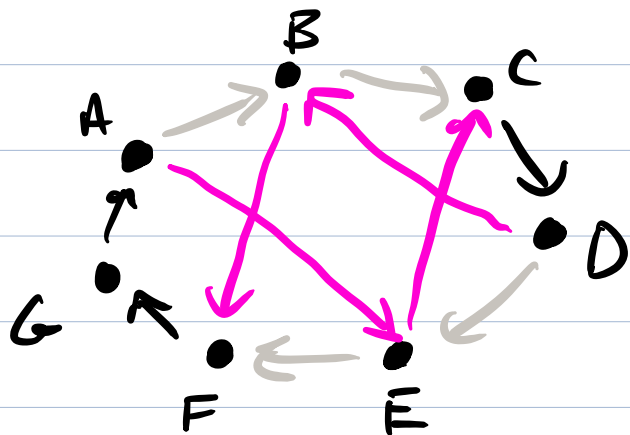

Let d = distance function

$Score = d(A,B) + d(B,C)$
$\qquad + d(C,D) + d(D,E)$
$\qquad + d(E,F) + d(F,G)$
$\qquad + d(G,A)$

Swap B and E:

$A \to B \to C \to D \to E \to F \to G \to A$

$A \to E \to C \to D \to B \to F \to G \to A$

Four edges changed

$$\text{Score} = \cancel{d(A,B)} + \cancel{d(B,C)} + d(C,D) + \cancel{d(D,E)} + \cancel{d(E,F)}$$

with annotations: $d(A,E)$, $d(E,C)$ over the first crossed terms, and $d(D,B)$, $d(B,F)$ over the later crossed terms.

$$+ d(F,G) + d(G,A)$$
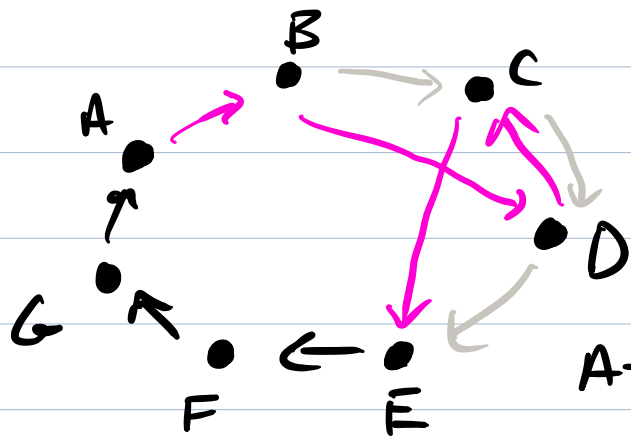
Even if you have 1000 cities, only 4 edges change.

New score = old score − 4 edges + 4 edges

8 distances instead of 300 $= \dfrac{300}{8} = 37.5x$

Is swapping 2 cities a good tweak?
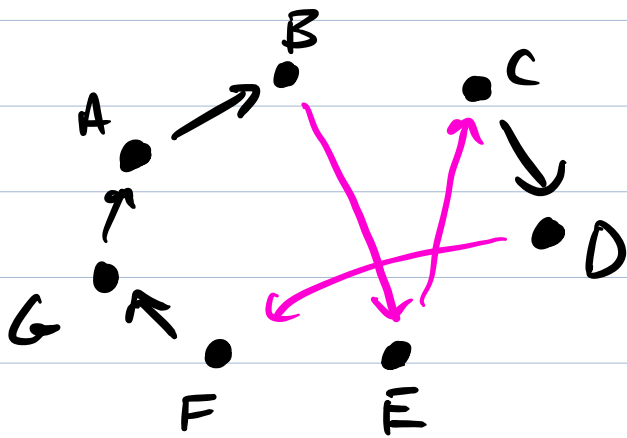No. The reason is that smaller tweaks are better than larger tweaks.

To change 2 edges:

$$A \to B \to D \to C \to E \to F \to G$$
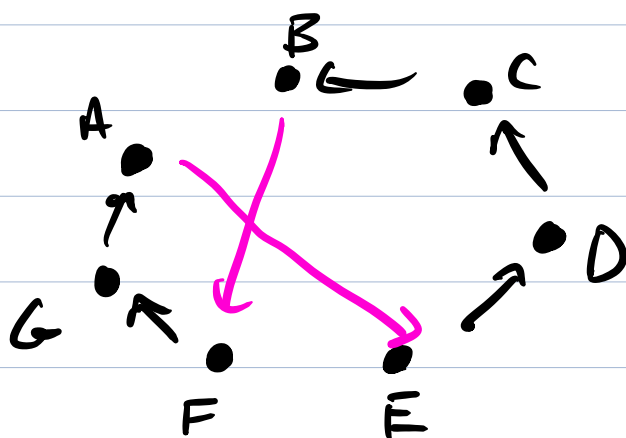
swapped two consecutive cities

$A \rightarrow B \rightarrow C \rightarrow D \dashrightarrow E \rightarrow F \rightarrow G \rightarrow A$

$A \rightarrow B \rightarrow E \rightarrow C \rightarrow D \rightarrow F \rightarrow G \rightarrow A$

$A \rightarrow \boxed{B \rightarrow C \rightarrow D \dashrightarrow E} \rightarrow F \rightarrow G \rightarrow A$

$A \rightarrow E \rightarrow D \rightarrow C \rightarrow B \rightarrow F \rightarrow G \rightarrow A$

"Reverse a block"

Demos: 05 - Steepest Asc. reverse block 50 6.48
        06 -   "   "   "   "   300 14.36

How can we adapt this for continuous
spaces? Check n things in the neighborhood
and move to the best.

MH #3   n-Trial Steepest Ascent

```
x = random element of S
while True:
    temp = x
    repeat n times:
        s = tweak(x)
        if score(s) > score(temp):
            temp = s
    x = temp
```

Later: good ways to tweak for continuous
          spaces

When n=1, this is called "Hill Climbing".
   Try a random neighbor, if it's better, move there.

MH #4: Hill Climbing
```
x = random element of S
while True:
    s = tweak(x)
    if score(s) > score(x):
        x = s
```

Demos:

07 -	TSP	HC	Swap 2	50	8.42
08 -	TSP	HC	Swap 2	300	29.4
09 -	TSP	HC	Rev. Bl.	50	6.72
10 -	TSP	HC	Rev. Bl.	300	14.62

None of these four ever allow a _worse_
score. You must always move uphill.

MHs = "the art of going downhill smartly"

exploration   vs.   exploitation
diversification vs.   intensification

↓   ↘
Looking in areas of   Searching the area
the search space that   you're already in for
you haven't seen before better and better solutions

Maximally exploitative: Hill climbing
Maximally explorative: Random search

We want things more in the middle.
One way: random restarts
   * Any H-C metaheuristic, runs for a while,
     stops, restarts at a new random
     point.