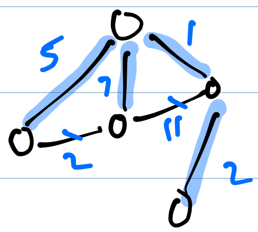
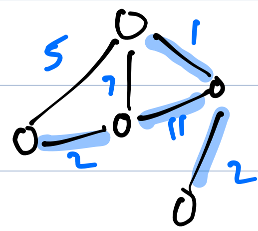
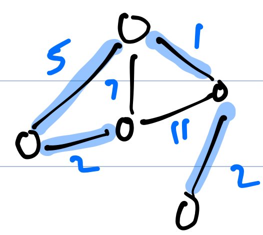
MSSC 6000
Feb 11, 2022 – Day 9

Lecture 3 - Greedy Algorithms (continued)

Minimum Spanning Tree Problem:
Given a weighted graph G, find the subset of edges that forms a minimum-weight tree that touches all of the vertices







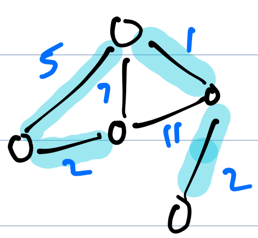
5+7+1+2 = 15

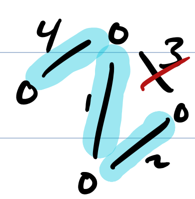
1+2+2+11 = 16

1+2+2+5 = 10
Minimal!

Possible Greedy Algorithms:

* start with no edges, and at each
(A) point add the cheapest edge that does not make a cycle.



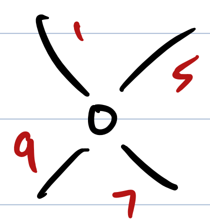


(B)

* Pick one node as start, and repeatedly choose the cheapest edge that connects to node you have reached so far, as long as it does not make a cycle

* best node = most or least connected (okay, but ignores weights)

* weight of a node = sum of weights of edges adjacent to it



best edge = cheapest edge on best node

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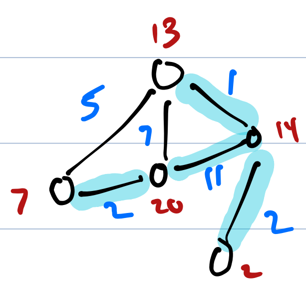
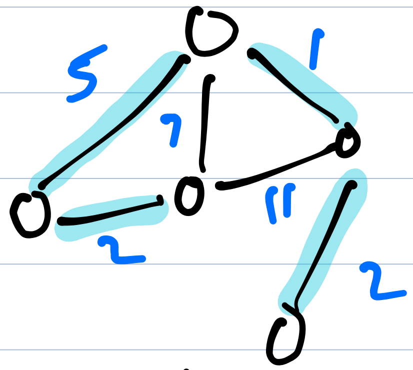


1+2+2+11 = 16

* start with all edges in your solutions, and repeatedly delete the most expensive

(c) one as long as doing so doesn't disconnect the graph



$5+2+2+1=10$

With our example graph, (A), (B), and (C) all gave the same solution, but this is not true in general.

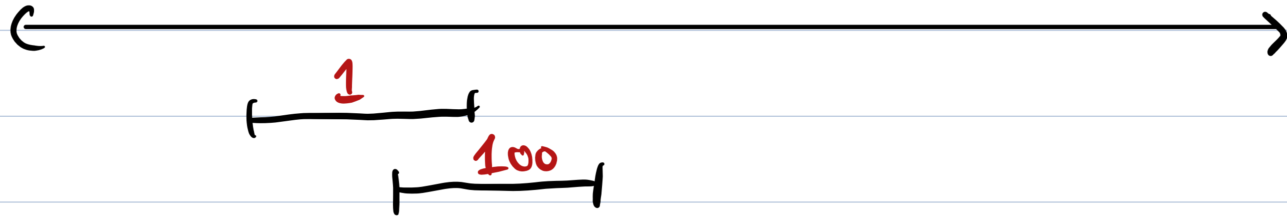
More important: Are any of these guaranteed to be optimal?

<u>Theorem</u>: All three of these greedy algos ((A), (B), (C)) are guaranteed to be optimal!

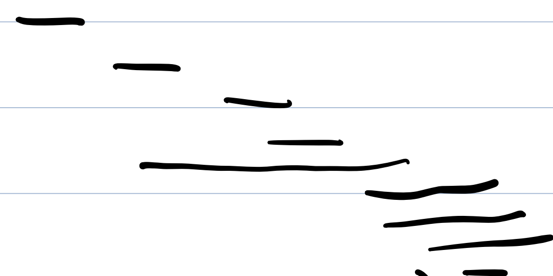
<u>Problem #3</u>: Weighted Interval Scheduling

This is like regular interval scheduling except each request r_i comes with a value v_i and your goal is to maximize the sum of the values of the requests you pick.

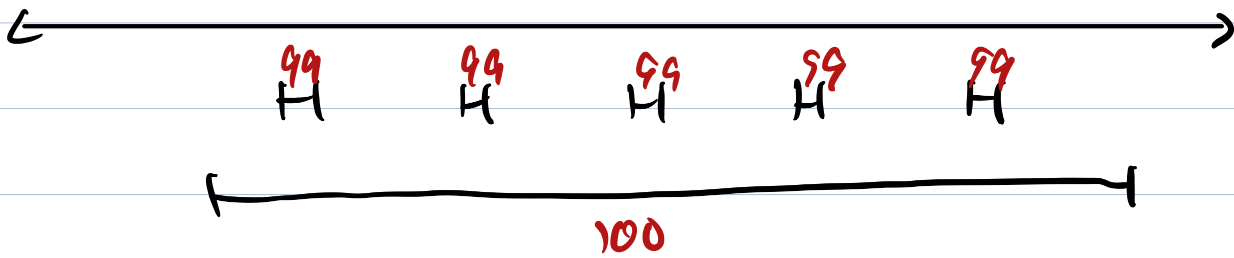
Our previous greedy algo (best = earliest end) is bad now.



Possible greedy algos:



* best = most value



* best = (highest value among [earliest end time and all that conflict with it])

* best = sort by earliest end, take longest prefix without conflicts, plus 1 more
take any that don't conflict at all

choose between two remaining possibilities

* best = shortest

* best = maximizes $\frac{\text{value}}{\text{duration}}$ (value density)

None of these are optimal!