MSSC 6000
Feb 6, 2022

Announcements:
→ HW 1 due next Wed on D2L

Lecture 3 - Greedy Algorithms (continued)

Proof: Let R be a set of requests.
Let A be the output solution from our greedy algorithm.
Let $\mathcal{O}$ be an arbitrary optimal solution.
We want to show $|A| = |\mathcal{O}|$.

$|A| \leq |\mathcal{O}|$ is obvious because we assumed $\mathcal{O}$ was optimal.
So we need to show $|A| \geq |\mathcal{O}|$.
Suppose the requests in A are:
$A = \{(s_1, f_1), (s_2, f_2), \ldots, (s_k, f_k)\}$
and in $\mathcal{O}$:
$\mathcal{O} = \{(s_1', f_1'), (s_2', f_2'), \ldots, (s_m', f_m')\}$
and assume we have listed them in order:

$s_1 < f_1 \le s_2 < f_2 \le \cdots$
$s_1' < f_1' \le s_2' < f_2' \le \cdots$

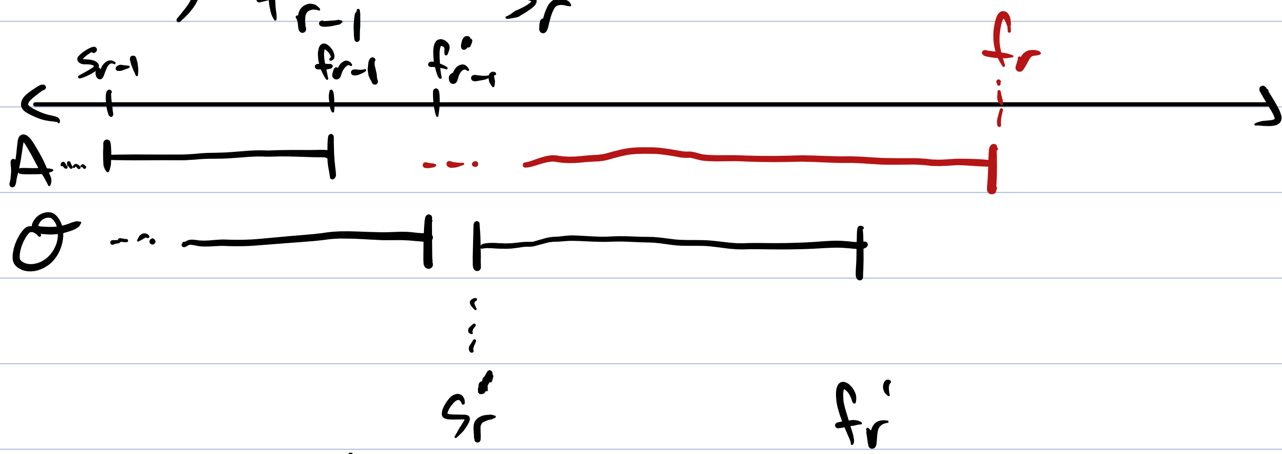
Note that $k \le m$.

We'll now show that A "stays ahead" of $\mathcal{O}$:
$$f_r \le f_r', \quad \text{for } r = 1, \ldots, k.$$
In English, the r^{th} meeting of A finishes before the r^{th} task of $\mathcal{O}$.

We'll prove this by induction.
Base case: $r=1$, $f_1 \le f_1'$
This is true because we defined our greedy algo. to start by picking the earliest ending time.

<u>Induction Step:</u>
Assume $f_{r-1} \le f'_{r-1}$.
We want to show $f_r \le f'_r$.
We know:
* $f_{r-1} \le f'_{r-1}$ (induction hypothesis)
* $f'_{r-1} \le s_r'$ (otherwise $\mathcal{O}$ would have conflicting meetings)

$\Rightarrow f_{r-1} \leq s_r'$



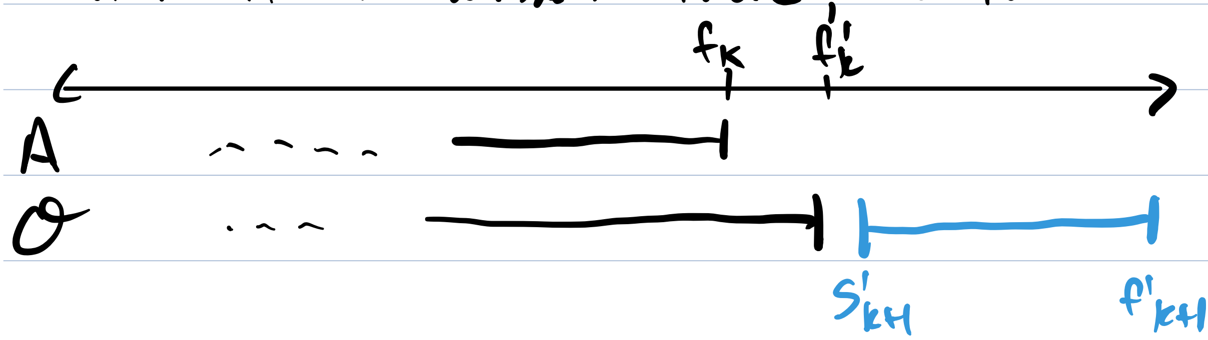
If it's not true that $f_r \leq f_r'$, then we have the red request above. This would never happen because the greedy algo would have picked (s_r', f_r') instead (s_r, f_r). So, the induction is done and we know that

$$f_r \leq f'_r \quad \text{for all } r \in \{1, 2, \ldots, k\}.$$

So, we know A "stays ahead" of O, and we want to show that $|O|$ is not bigger than $|A|$.

$|O| \leftarrow m$, $|A| \leftarrow k$

What if it wasn't true, what if $|O| > |A|$.



This wouldn't happen because the greedy algo would have picked (s'_{k+1}, f'_{k+1}).

$\blacksquare$

-> Python Tip of the Day

-> Code our greedy algorithm.