MATH 4670 / 5670 – COMBINATORICS
HOMEWORK 5

Spring 2022

due Wednesday, April 27, by the beginning of class

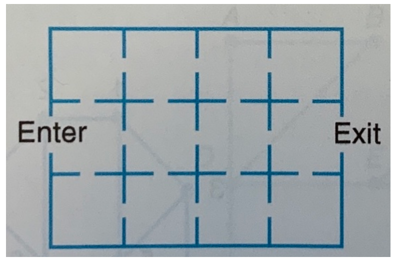
This homework assignment was written in LaTeX. You can find the source code on the course website.

Mathematical Writing: An important component of this course is learning how to write mathematics correctly and concisely. Your goal should always be the convince the reader that you are correct! That means explaining your thinking and each step in your solution. I expect you to do the following: explain your reasoning, don't leave out steps, and use full sentences with correct spelling and grammar (including your use of math symbols).

All answers must be fully justified to receive credit. Answers without justification will not be considered correct.

1. See the graph G in Figure 5.46 on page 102 (in Section 5.9, Exercises) of the free textbook "Applied Combinatorics" by Keller and Trotter.
   (a) What is the degree of vertex 8?
   (b) What is the degree of vertex 10?
   (c) How many vertices of degree 2 are there in G? List them.
   (d) Find a cycle of length 8 in G.
   (e) What is the length of a shortest path from 3 to 4?
   (f) What is the length of a shortest path from 8 to 7?
   (g) Find a path of length 5 from vertex 4 to vertex 6.
2. Draw a graph with 8 vertices, all of odd degree, that does not contain a path of length 3, or explain why such a graph does not exist.
3. Draw a graph with 6 vertices having degrees 5, 4, 4, 2, 1, and 1 or explain why such a graph does not exist.
4. Let $T = (V, E)$ be a tree. Prove that adding any additional edge to T (connecting two vertices that already exist in T) must create a cycle.
5. Let $T = (V, E)$ be a tree. Prove that deleting any single edge of T will cause T to become disconnected.
6. Prove that any connected graph with n vertices must have at least $n - 1$ edges. (In other words, it is not possible to build a connected graph with n vertices and $n - 2$ edges or fewer.)
7. We proved in class that every tree with n vertices must have exactly $n - 1$ edges. In the exercise, I want you to prove a version of the converse: Suppose G is a connected graph with n vertices and $n - 1$ edges. Then G must be a tree.

8. The floorplan below shows the Boatsville College Museum of Art. Is it possible for a guest to visit the entire museum by going through every doorway once and only once? (Convert the floorplan to a graph.) Make sure you justify your answer!



9. For the graph below, find an Eulerian path (if one exists), or explain why there can't be one (if one doesn't exist). Do the same thing for an Eulerian circuit. *When giving a path or circuit, be sure to find a way to present it on paper that makes sense!*