Monday, March 29 Lecture #27

Simulated Annealing

Acceptance Conditions:

Temp T

sol x

$s = \text{tweak}(x)$

$\Delta = \text{score}(s) - \text{score}(x)$

If $\Delta \geq 0$, accept.

If $\Delta \leq 0$, accept with prob $p = e^{\Delta/T}$

Cooling Schedule:

* Geometric (most common): ($\alpha = 0.99$, 0.999, 0.95)

For some α

$T \leftarrow T \cdot \alpha$

Ex: Initial temp $T_0 = 10$, $\alpha = 0.9$

$10 \rightarrow 9 \rightarrow 8.1 \rightarrow 7.29 \rightarrow \ldots$

(we'll never hit 0)

Formula: $T_n = T_0 \cdot \alpha^n$

* Linear: $T \leftarrow T - \beta$

$T_n = T_0 - \beta \cdot n$

* Many more, even some that are not always decreasing

Process:

Pick an initial temperature T (How?)

x = random solution

best = x

Repeat: (How long?)

For a while: (How long?)

s = tweak(x)

Δ = score(s) - score(x)

if $\Delta \geq 0$:

x = s

if score(x) > score(best):

best = x

else: ($\Delta < 0$)

r = random # in [0,1]

if $r < e^{\Delta/T}$: → 0.7

x = s

adjust the temperature according to the cooling schedule

return best

Picking the initial temp

First pick p_0: the initial prob. with which you want worsening moves to be accepted.

Lots of schools of thought: depends on the problem.

↳ landscape with lots of little hills? fewer huge hills?

Typical values for p_0 are 0.9, 0.5, 0.2

How can we find the temp T that leads to p_0? Recall:

$$p = e^{\Delta / T}$$

We don't know Δ.

We will approximate the average value of Δ, then use that to calculate T.

```
trials = []
while len(trials) < 1000:
    x = random solution
    s = tweak(x)
    if score(s) < score(x):
        trials.append(score(s) - score(x))
avg = sum(trials) / len(trials)
```

median

an approx. for the average value of Δ.

$$p = e^{\Delta/T} \Rightarrow \ln(p) = \Delta/T \Rightarrow T = \frac{\Delta}{\ln(p)}$$

This might work, but might fail:
Tweaks from totally random solutions might not be representative of tweaks after exploring for a while.

How long do we run at a fixed temp before moving to the next temp?

(art)
* N tweak attempts in total
* K worsenings rejected

or

L worsenings accepted, which ever comes first

$L = 1000$, $K = 100{,}000$

How long do we cool before stopping?

* Run out of time/patience
* No worsening moves accepted for some amount of time

(at each step, you could print the % of worse moves accepted)

* Pre-set end temperature

$$T_f = 0.001 \cdot T_0$$