

Monday, March 29

Lecture #27

Simulated Annealing

Acceptance Conditions:

Temp T

sol x

$s = \text{tweak}(x)$

$\Delta = \text{score}(s) - \text{score}(x)$

If $\Delta \geq 0$, accept.

If $\Delta \leq 0$, accept with prob

$$p = e^{\Delta/T}$$

Cooling Schedule:

* Geometric (most common): $(\alpha = 0.99)$
For some α

$$T \leftarrow T \cdot \alpha$$

Ex: Initial temp $T_0 = 10$, $\alpha = 0.9$

$10 \rightarrow 9 \rightarrow 8.1 \rightarrow 7.29 \rightarrow \dots$

(we'll never hit 0)

Formula: $T_n = T_0 \cdot \alpha^n$

* Linear: $T \leftarrow T - \beta$

$$T_n = T_0 - \beta \cdot n$$

* Many more, even some that are not always decreasing

Process:

Pick an initial temperature T

(How?)

$x =$ random solution

best = x

Repeat:

(How long?)

For a while:

(How long?)

$s =$ tweak(x)

$\Delta =$ score(s) - score(x)

if $\Delta \geq 0$:

$x = s$

if score(x) > score(best):

best = x

else:

($\Delta < 0$)

$r =$ random # in $[0, 1]$

if $r < e^{\Delta/T}$:

→ 0.7

$x = s$

adjust the temperature according to the cooling schedule

return best

Picking the initial temp

First pick p_0 : the initial prob. with which you want worsening moves to be accepted.

Lots of schools of thought: depends on the problem.

↳ landscape with lots of little hills?
fewer huge hills?

Typical values for p_0 are 0.9, 0.5, 0.2

How can we find the temp T that leads to p_0 ? Recall:

$$p = e^{-\Delta/T}$$

We don't know Δ .

We will approximate the average value of Δ , then use that to calculate T .

trials = []

while len(trials) < 1000:

x = random solution

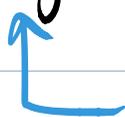
s = tweak(x)

if score(s) < score(x):

trials.append(score(s) - score(x))

avg = sum(trials) / len(trials)

median



an approx. for the average
value of Δ .

$$p = e^{-\Delta/T} \Rightarrow \ln(p) = -\Delta/T \Rightarrow T = \frac{\Delta}{-\ln(p)}$$

This might work, but might fail:

Tweaks from totally random solutions
might not be representative of tweaks
after exploring for a while.

How long do we run at a fixed temp
before moving to the next temp?

(art) * N tweak attempts in total
* K worsenings rejected

or

L worsenings accepted,
which ever comes first
 $L=1000, K=100,000$

How long do we cool before stopping?

- * Run out of time/patience
- * No worsening moves accepted for some amount of time
(at each step, you could print the % of worse moves accepted)
- * Pre-set end temperature

$$T_f = 0.001 \cdot T_0$$