MATH 1450 – EXAM 3

Friday, April 30

Name: Key

Read these instructions carefully before beginning.

1. You have 50 minutes to complete this exam, and then 15 minutes to scan and upload your work to D2L.
2. **You are permitted to use your textbook (physical or digital copy) and any lecture notes YOU took this semester.** You are not permitted to use any other resources, including a calculator, your graded work, the internet, notes anyone else took, other people, etc.
3. You must show work and explain all reasoning unless otherwise stated.
4. If you want to write directly on the pdf that is fine. Otherwise you can work on blank paper.
5. You must work neatly and clearly, from the top to the bottom of the page, with the questions in the correct order. For example, do not do Q1 and Q2 on the left half of a page, then do Q3 up in the top right corner.
6. You do not need to rewrite the questions, but you must make sure your answers are correctly numbered.
7. If I cannot read your writing, you will not receive credit.
8. **Your work MUST be submitted as a single pdf file containing nicely cropped, well-lit pictures of your work. I previously sent instructions for one possible app to do this.**

The Marquette University honor code obliges students:

- To fully observe the rules governing exams and assignments regarding resource material, electronic aids, copying, collaborating with others, or engaging in any other behavior that subverts the purpose of the exam or assignment and the directions of the instructor.
- To turn in work done specifically for the paper or assignment, and not to borrow work either from other students, or from assignments for other courses.
- To complete individual assignments individually, and neither to accept nor give unauthorized help.
- To report any observed breaches of this honor code and academic honesty.

Section 1: True / False. Choose True or False. **If you choose False, explain briefly why the statement is wrong.**

3pts

1. The derivative of $\sin(x)$ is $\cos(x)$ and the derivative of $\cos(x)$ is $\sin(x)$.

True False

The derivative of $\cos(x)$ is $-\sin(x)$.

3

2. If f is a continuous function on a closed interval $[a, b]$, then f is guaranteed to have a global maximum and a global minimum.

True False

Fact from 4.2.

3

3. L'Hopital's rule can be applied directly to the forms $0 \cdot \infty$ and $0 \cdot (-\infty)$.

True False

It cannot be applied to these forms.

3

4. The linear approximation of a function $g(x)$ at a point $x = a$ is good for x-values close to a, but usually gets worse as x gets further away from a.

True False

12

Section 2: Short Response. You **MUST** show your work for these questions.

5

5. Compute the derivative of $\sin(\ln(x) + x)$.

$$\frac{d}{dx}\left(\sin(\ln(x)+x)\right) = \underbrace{\cos(\ln(x)+x)\cdot\frac{d}{dx}(\ln(x)+x)}^{\text{chain rule}}$$

$$= \cos(\ln(x)+x)\cdot\left(\frac{1}{x}+1\right)$$

5

6. Find the linear approximation of $\ln(x^2 + 1)$ at $x = 3$.

$f(x) = \ln(x^2+1)$ $\quad f(3) = \ln(10)$

$f'(x) = \dfrac{2x}{x^2+1}$ $\quad f'(3) = \dfrac{6}{10} = \dfrac{3}{5}$

$$y = \frac{3}{5}(x-3) + \ln(10)$$

$$= \frac{3}{5}x + \left(\ln(10) - \frac{9}{5}\right)$$

(5)

7. Compute the limit using any method you like, but be sure to justify your answer.

$$\lim_{x\to 0} \frac{e^{2x} - 2x - 1}{x^2} = \frac{0}{0}$$

By L'H, $= \lim_{x\to 0} \frac{2e^{2x}-2}{2x} = \frac{0}{0}$

By L'H, $= \lim_{x\to 0} \frac{4e^{2x}}{2} = 2$

(5)

8. Find all critical points of the function $f(x) = \tan(x)$ on the interval $[-2\pi, 2\pi]$. (You may use the unit circle given in a later question).

$f'(x) = \frac{1}{\cos^2(x)}$.

C.P.s where $f'(x)=0$ or [$f'(x)$ undefined but $f(x)$ defined].

$f'(x)$ never $=0$ because the numerator cannot be made 0.

$f'(x)$ undef. when $\cos(x)=0$, but $f(x)$ also undef. at those points.

So, there are <u>no</u> critical points.

(20)

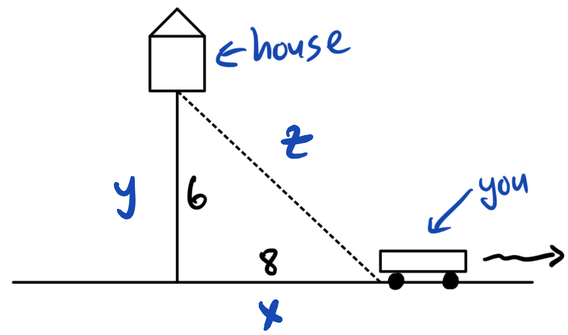
Section 3: Free Response. Answer each question, **showing all work**.

9pts

9. You are driving on a straight highway at 60 miles per hour. There is a house whose distance from the highway is 6 miles, and you are 8 miles past the point in the highway closest to the house. (See the figure below.) At this moment, what is the rate of change of the distance between you and house (the dashed line)?

Known: $x = 8$ mi. $\frac{dx}{dt} = 60$ mi/hr

$y = 6$ mi $\frac{dy}{dt} = 0$ mi/hr



Want $\frac{dz}{dt}$. Equation relating the quantities:

$$x^2 + y^2 = z^2.$$

Take the derivative with respect to t.

$$2x \cdot \frac{dx}{dt} + 2y \cdot \frac{dy}{dt} = 2z \frac{dz}{dt}$$

* = known

* = want

Need z:

$$8^2 + 6^2 = z^2$$

$$\Rightarrow 100 = z^2$$

$$\Rightarrow z = 10$$

So, $\cancel{2} \cdot 8 \cdot \overset{6}{\cancel{60}} + \cancel{2} \cdot 6 \cdot 0 = \cancel{2} \cdot \cancel{10} \cdot \frac{dz}{dt}$

$$\Rightarrow \boxed{\frac{dz}{dt} = 48 \text{ mi/hr}}$$

10pts 10. Consider the function $p(\theta) = \sin^2(\theta) + \cos(\theta)$, on the closed domain $[0, 2\pi]$. You may refer at any point to the unit circle at the bottom of this page.

(4) (a) Find all critical points of $p(\theta)$ (only in the given domain $[0, 2\pi]$).

$$p'(\theta) = 2\sin(\theta)\cos(\theta) - \sin(\theta) = \sin(\theta) \cdot (2\cos(\theta) - 1).$$

When is $p'(\theta) = 0$? When $\sin(\theta) = 0 \Rightarrow \theta = 0, \pi, 2\pi$

on boundary

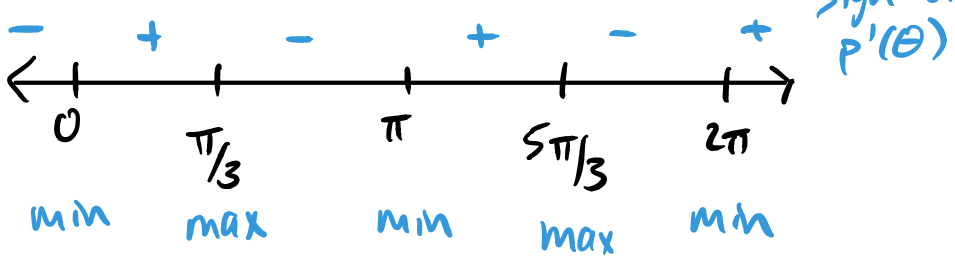
or

$2\cos(\theta) - 1 \Rightarrow \cos(\theta) = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{3}, \frac{5\pi}{3}$

$p'(\theta)$ is never undefined, so no critical points from that.

(3) (b) Identify which of the critical points are local maxima, which are local minima, and which are neither, using any method you want.

First derivative test



(3) (c) What are the global maximum and global minimum of $p(\theta)$ in the given domain?

$p(\theta) = \sin^2(\theta) + \cos(\theta)$

$p(0) = 0 + 1 = 1$

$p(\pi) = 0 + (-1) = -1$ global min

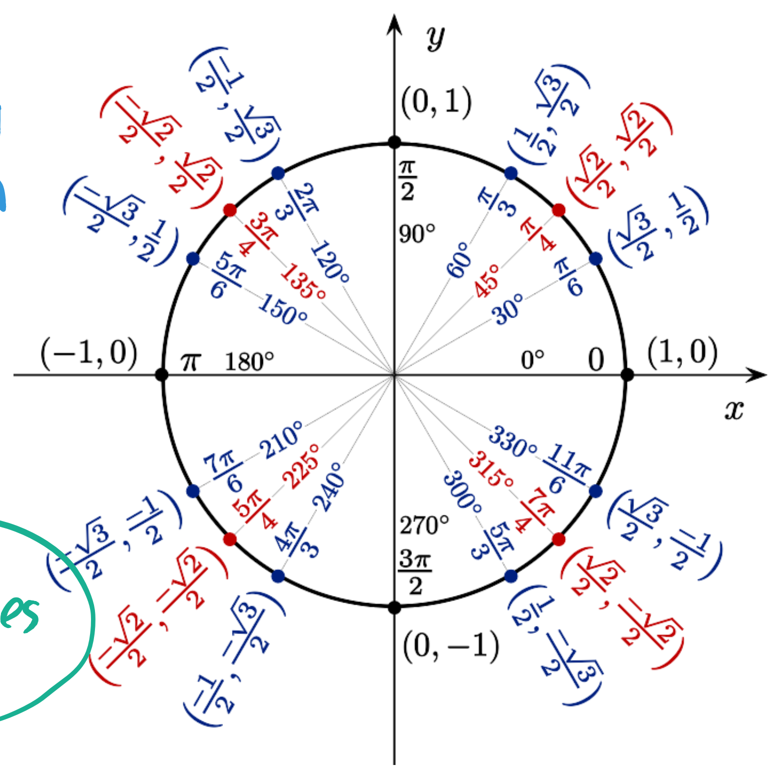
$p(2\pi) = 0 + 1 = 1$

$p(\pi/3) = \left(\frac{\sqrt{3}}{2}\right)^2 + \frac{1}{2}$

$= \frac{3}{4} + \frac{1}{2} = 1.25$

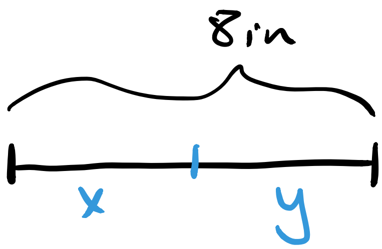
2 global maxes

$p\left(\frac{5\pi}{3}\right) = \left(-\frac{\sqrt{3}}{2}\right)^2 + \frac{1}{2} = 1.25$

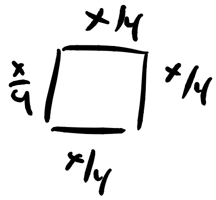


9 pts 11. You are given a piece of wire that is 8 inches long. Your instructions are to cut the wire at some point giving you two pieces, then bend each piece into a square. (You are also allowed to not cut it at all and only make one square, which is like the first piece having length 0.)

What lengths should you cut the wire into to maximize the area of the squares you form?



$x+y=8$

Bend x into a square: 

area $= \left(\frac{x}{4}\right)^2 = \frac{x^2}{16}$

Bend y into a square: area $= \frac{y^2}{16}$.

Total area $\frac{x^2+y^2}{16}$, but subject to the constraint $x+y=8$.

$\Rightarrow y = 8-x$

$$\frac{x^2+y^2}{16} = \frac{x^2+(8-x)^2}{16} = \frac{x^2+(64-16x+x^2)}{16}$$

$$= \frac{2x^2-16x+64}{16}$$

Call this $f(x)$. We want the global max on $[0,8]$.

$f'(x) = \frac{4x-16}{16} \Rightarrow f'(x)=0$ when $x=4$. ← critical pt.

Need to check f at critical point and endpoints.

$f(0)=4$, $f(4)=2$, $f(8)=4$. So global max at $x=8, y=0$ and $x=0, y=8$.