Next to each question, I wrote an estimate of the amount of time it should take if you have prepared adequately.
Total: 35 mins

MATH 1450 – EXAM 2

Wednesday, March 31

Name: Key

<u>Read these instructions carefully before beginning.</u>

1. You have 50 minutes to complete this exam, and then 15 minutes to scan and upload your work to D2L.
2. **You are permitted to use your textbook (physical or digital copy) and any <u>lecture</u> notes <u>YOU</u> took this semester.** You are not permitted to use any other resources, including a calculator, your graded work, the internet, notes anyone else took, other people, etc.
3. You must show work and explain all reasoning unless otherwise stated.
4. If you want to write directly on the pdf that is fine. Otherwise you can work on blank paper.
5. You must work neatly and clearly, from the top to the bottom of the page, with the questions in the correct order. For example, <u>do not</u> do Q1 and Q2 on the left half of a page, then do Q3 up in the top right corner.
6. You do not need to rewrite the questions, but you must make sure your answers are correctly numbered.
7. If I cannot read your writing, you will not receive credit.
8. **Your work MUST be submitted as a single pdf file containing nicely cropped, well-lit pictures of your work. I previously sent instructions for one possible app to do this.**

The Marquette University honor code obliges students:

- To fully observe the rules governing exams and assignments regarding resource material, electronic aids, copying, collaborating with others, or engaging in any other behavior that subverts the purpose of the exam or assignment and the directions of the instructor.
- To turn in work done specifically for the paper or assignment, and not to borrow work either from other students, or from assignments for other courses.
- To complete individual assignments individually, and neither to accept nor give unauthorized help.
- To report any observed breaches of this honor code and academic honesty.

Time **Section 1: True / False.** Choose True or False. **If you choose False, explain briefly why the statement is wrong.**

1

1. The derivative of e^3 is e^3.

True　　　　False

e^3 is a constant, so its derivative is 0.

2

2. Suppose $f(x)$ and $g(x)$ are functions that are always decreasing. Then, $f(g(x))$ is always increasing.

True　　　　False

means $f'(x) < 0$ and $g'(x) < 0$ for all x

$\frac{d}{dx}(f(g(x))) = \underbrace{f'(g(x))}_{<0}\underbrace{g'(x)}_{<0}$

> 0

2

3. If $h(x)$ is an even function then $h'(x)$ is an odd function.

True　　　　False

$h(x)$ even $\Rightarrow h(-x) = h(x)$.

So, $h'(x) = (h(-x))' = -h'(-x)$ by the chain rule

$\Rightarrow h'(x)$ odd (or just think about the pictures)

1

4. Consider a car moving in one direction that we will consider the positive direction. If the car is moving faster and faster over time, then its acceleration must always be increasing.

True　　　　False

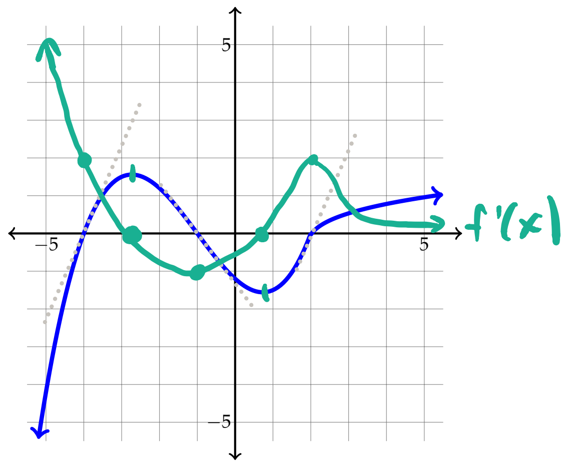
acceleration always positive, not necessarily increasing

6

(2)

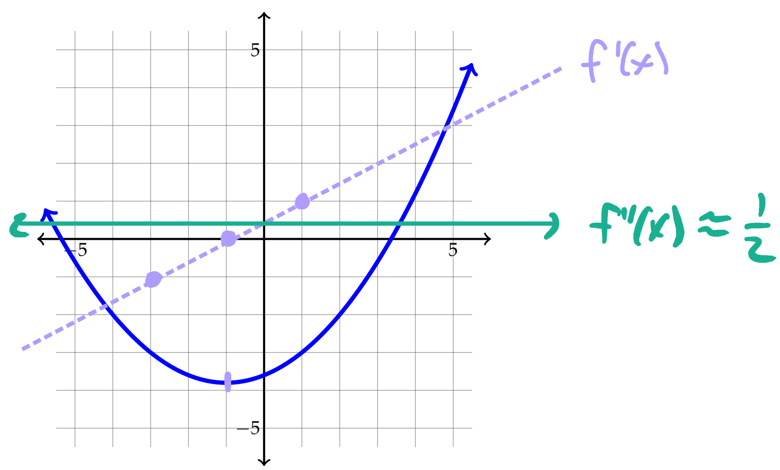
Section 2: Short Response. You do not need to show your work for these questions.

5. On the axes below, $f(x)$ is shown. Draw $f'(x)$ on the same axes. If you're writing answers on your own paper **redraw $f(x)$ with a dashed line or different color before drawing $f'(x)$, and make sure it's clear which function is which**.



(2)

6. On the axes below, $f(x)$ is shown. Draw the **second derivative** $f''(x)$ on the same axes. If you're writing answers on your own paper **redraw $f(x)$ with a dashed line or different color before drawing $f''(x)$, and make sure it's clear which function is which**.



(4)

7. Suppose that $f(x)$ and $g(x)$ are two functions, and that we know: $f(3) = 4$, $g(3) = 3$, $f'(3) = -2$, $g'(3) = 7$. Use this information to compute each of the following quantities if possible. If it's not possible, say so.

1 (a) $h'(3)$ where $h(x) = f(x) - g(x)$

$$h'(3) = f'(3) - g'(3) = -2 - 7 = -9$$

1 (b) $h'(3)$ where $h(x) = f(x)g(x)$

$$h'(3) = f'(3)g(3) + f(3)g'(3) = (-2)(3) + (4)(7) = 22$$

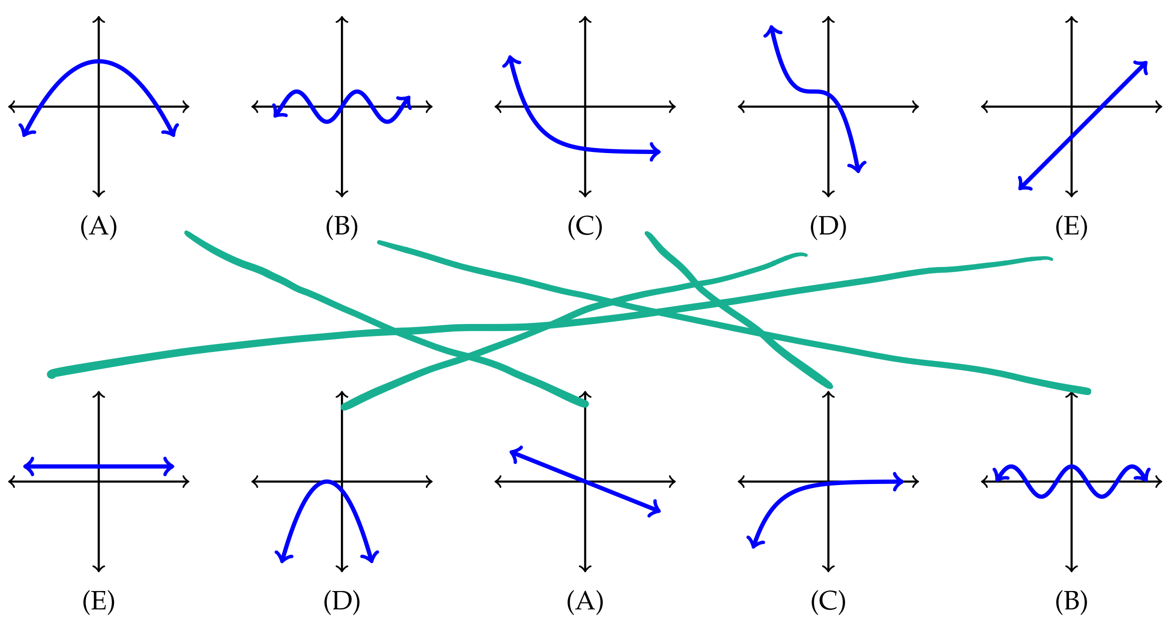
2 (c) $h'(3)$ where $h(x) = \dfrac{f(x)}{g(x)}$

$$h'(3) = \frac{g(3)f'(3) - f(3)g'(3)}{(g(3))^2} = \frac{(3)(-2) - (4)(7)}{(3)^2} = -34/9$$

2 (d) $h'(3)$ where $h(x) = f(g(x))$

$$h'(3) = f'(g(3))g'(3) = f'(3)g'(3) = -14$$

2 8. Match each of the functions (A) - (E) with their correct derivatives (1) - (5).



8

Section 3: Free Response. Answer each question, **showing all work**.

9. While standing on a platform, you throw a balloon up into the air. It's height in feet above the ground t seconds after being thrown is $h(t) = -t^2 + 10t + 11$.

1

(a) What is the velocity of the balloon the moment you release it at $t = 0$?

$h'(t) = -2t + 10$

$h'(0) = 10$ ft/sec

2

(b) At what time does the balloon reach its highest point above the ground, and what is that height?

highest point $\Rightarrow$ velocity $= 0$ (turning around)

$\Rightarrow -2t + 10 = 0$

$\Rightarrow$ $t = 5$ seconds

height $= h(5) = -25 + 50 + 11 =$ 36 feet

2

(c) At what time does the balloon hit the ground?

height $= 0 \Rightarrow -t^2 + 10t + 11 = 0$

$\Rightarrow -(t^2 - 10t - 11) = 0$

$\Rightarrow -(t - 11)(t + 1) = 0$

$\Rightarrow t = 11$ seconds

1

(d) What is the velocity of the balloon when it hits the ground?

$h'(11) = -2(11) + 10 = -12$ ft/sec

6

10. Define $A(x) = 3x^2$. (Note: you should not be using the power rule anywhere in this answer.)

1

(a) Write down the limit definition of the derivative of $A(x)$.

$$\lim_{h \to 0} \frac{A(x+h) - A(x)}{h}$$

$$= \lim_{h \to 0} \frac{3(x+h)^2 - 3x^2}{h}$$

3

(b) Solve this limit algebraically to find the derivative.

$$\lim_{h \to 0} \frac{3(x+h)^2 - 3x^2}{h} = \lim_{h \to 0} \frac{3(x^2 + 2xh + h^2) - 3x^2}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{3x^2} + 6xh + 3h^2 - \cancel{3x^2}}{h}$$

$$= \lim_{h \to 0} (6x + 3h) = 6x$$

4

11. Compute the derivative of $f(x) = \frac{\sqrt{x}+x}{2^x}$.

Quotient Rule: $a(x) = \sqrt{x} + x$ $\quad a'(x) = \frac{1}{2}x^{-1/2} + 1$

$b(x) = 2^x$ $\quad b'(x) = \ln(2) \cdot 2^x$

$$f'(x) = \frac{b(x)a'(x) - a(x)b'(x)}{(b(x))^2}$$

$$= \frac{2^x\left(\frac{1}{2}x^{-1/2} + 1\right) - \ln(2) \cdot 2^x \cdot (\sqrt{x} + x)}{(2^x)^2}$$

$$= \frac{\cancel{2^x}\left(\frac{1}{2}x^{-1/2} + 1 - \ln(2)(\sqrt{x} + x)\right)}{(2^x)^{\cancel{2}}}$$

$$= \frac{\frac{1}{2\sqrt{x}} + 1 - \ln(2)(\sqrt{x} + x)}{2^x}$$

4

12. Compute the derivative of $r(t) = \cancel{(xe^x + 10x)^8}$. $(te^t + 10t)^8$

Chain Rule: $f(t) = t^8 \qquad f'(t) = 8t^7$

$g(t) = te^t + 10t \qquad g'(t) = \underbrace{e^t + te^t}_{\text{from the product rule}} + 10$

$$r'(t) = f'(g(t)) \cdot g'(t)$$

$$= f'(te^t + 10t) \cdot (e^t + te^t + 10)$$

$$= 8(te^t + 10t)^7 \cdot (e^t + te^t + 10)$$

3