MATH 1450 – EXAM 1

Friday, February 26

Name: Key!

Read these instructions carefully before beginning.

1. You have 50 minutes to complete this exam, and then 15 minutes to scan and upload your work to D2L.
2. **You are permitted to use your textbook (physical or digital copy) and any lecture notes YOU took this semester.** You are not permitted to use any other resources, including a calculator, your graded work, the internet, notes anyone else took, other people, etc.
3. You must show work and explain all reasoning unless otherwise stated.
4. If you want to write directly on the pdf that is fine. Otherwise you can work on blank paper.
5. You must work neatly and clearly, from the top to the bottom of the page, with the questions in the correct order. For example, do not do Q1 and Q2 on the left half of a page, then do Q3 up in the top right corner.
6. You do not need to rewrite the questions, but you must make sure your answers are correctly numbered.
7. If I cannot read your writing, you will not receive credit.
8. **Your work MUST be submitted as a single pdf file containing nicely cropped, well-lit pictures of your work. I previously sent instructions for one possible app to do this.**

The Marquette University honor code obliges students:

- To fully observe the rules governing exams and assignments regarding resource material, electronic aids, copying, collaborating with others, or engaging in any other behavior that subverts the purpose of the exam or assignment and the directions of the instructor.
- To turn in work done specifically for the paper or assignment, and not to borrow work either from other students, or from assignments for other courses.
- To complete individual assignments individually, and neither to accept nor give unauthorized help.
- To report any observed breaches of this honor code and academic honesty.

Section 1: True / False. Choose True or False. **If you choose False, explain briefly why the statement is wrong.**

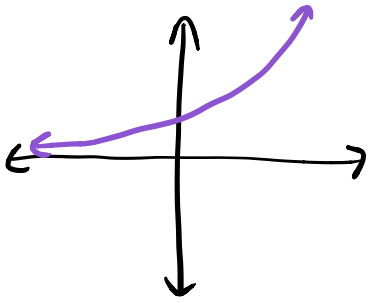
1. The degree of a polynomial is the number of coefficients it has.

True False

Example: $x^{10}+2$ has only two coefficients, but degree 10.

2. If a^x is an exponential growth function, then $\lim_{x\to-\infty} a^x = -\infty$.

True False



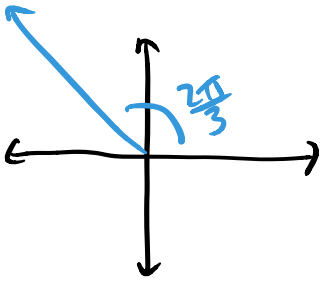
↖ Exp. growth

$\lim_{x\to-\infty} a^x = 0$, not $-\infty$

3. $\sin(2\pi/3)$ is positive.

True False

4. $\cos(2\pi/3)$ is positive.



True False

$\frac{2\pi}{3}$ is between $\frac{\pi}{2}$ and π

(because $\frac{2}{3}$ is between $\frac{1}{2}$ and 1)

cos represents the x-value, which is negative here

Section 2: Short Response. You do not need to show your work for these questions.

5. Listed below are seven different functions. Order them from largest to smallest in terms of their end behavior as $x \to \infty$ (for example, start with the one that dominates all of the others, then the next one is the one that dominates all remaining ones, etc).

x^2 $\cos(x)$ $\log_5(x)$ $\frac{x^{10}}{100}$ 1.5^x $\sqrt{x}$ $-x^3$

$\sqrt{x} = x^{1/2}$

positive power functions

stuck between -1 and 1

exponential

negative

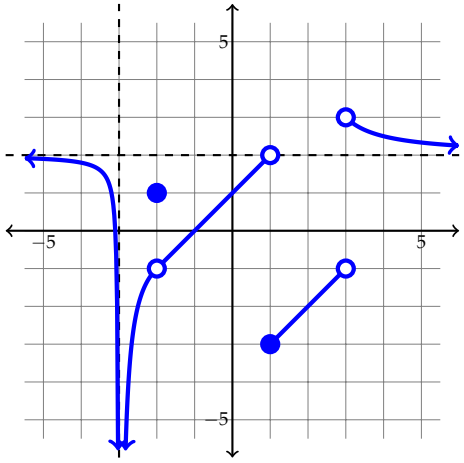
exp. > power > log, but all $\to \infty$

$\cos(x)$ finite

$-x^3 \to -\infty$

So, 1.5^x, $\frac{x^{10}}{100}$, x^2, $\sqrt{x}$, $\log_5(x)$, $\cos(x)$, $-x^3$

6. Based on the graph of $q(x)$ below, determine each quantity. If the limit is $+\infty$ or $-\infty$, write that (instead of "does not exist"). You do not need to explain your reasoning for this problem.



(a) $\lim_{x\to-\infty} q(x)$ 2 (hor. asymptote)

(b) $q(-2)$ 1

(c) $\lim_{x\to-2} q(x)$ -1

(d) $q(1)$ -3

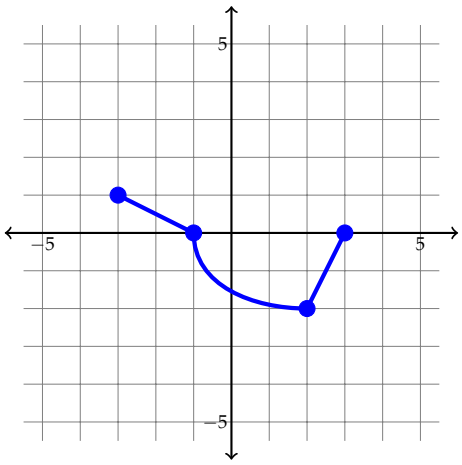
(e) $\lim_{x\to1} q(x)$ DNE

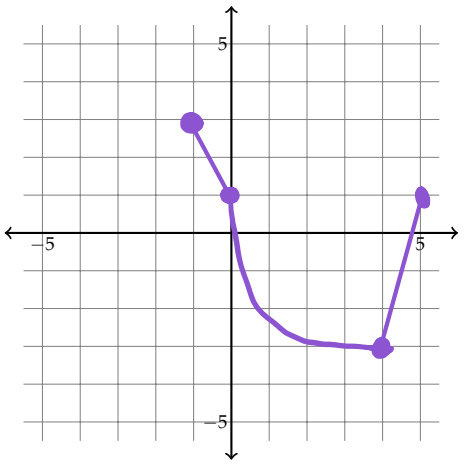
(f) $q(3)$ DNE

(g) $\lim_{x\to3} q(x)$ DNE

(h) $\lim_{x\to\infty} q(x)$ 2 (hor. asymptote)

7. The graph of the function $h(t)$ is show below. On the blank axes, draw the function $2h(t-2)+1$.





→ by 2
then vertical stretch ↕ by factor of 2
then ↑ by 1

Example: $(-3,1) \Rightarrow (-1,1) \Rightarrow (-1,2) \Rightarrow (-1,3)$

8. Match each of the functions below with its graph.

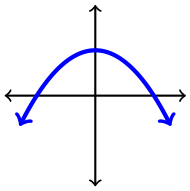
(I) $-(x+1)^3+1$

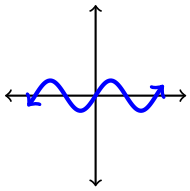
(II) $-x^2+3$

(III) $\sin(x)$

(IV) $x-2$

(V) $2^{-x}-3$


(A)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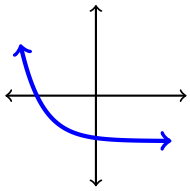
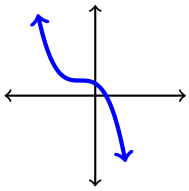

(C)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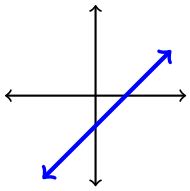

(E)

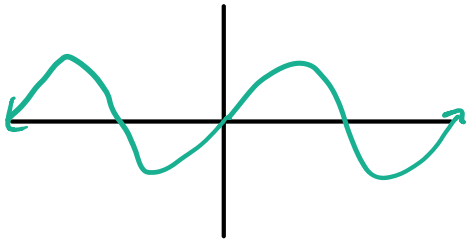
(II) (III) (V) (I) (IV)

Section 3: Free Response. Answer each question, **showing all work**.

9. Consider the two functions $f(x) = \sin(x)$ and $g(x) = x^2 - x + 3$.

   (a) Is $f(x)$ even, odd, or neither?

We know the graph of sin(x):



This has 180° degree rotational symmetry, so (ODD)

   (b) Is $g(x)$ even, odd, or neither?

$$g(-x) = (-x)^2 - (-x) + 3$$
$$= x^2 + x + 3$$

This is not the same as $g(x) \Rightarrow$ not even

This is not the same as $-g(x) \Rightarrow$ not odd

(neither)

   (c) What is $f(g(x))$?

$$f(g(x)) = f(x^2 - x + 3)$$
$$= \sin(x^2 - x + 3)$$

   (d) What is $g(f(x))$?

$$g(f(x)) = g(\sin(x))$$
$$= (\sin(x))^2 - \sin(x) + 3$$
$$(= \sin^2(x) - \sin(x) + 3)$$

10. (a) Expand and simplify the following expression as much as possible:

$$\log_{10}\left(\frac{10^S \cdot T^2}{A-B}\right)$$

$= \log_{10}(10^S \cdot T^2) - \log_{10}(A-B)$ (because $\log\left(\frac{x}{y}\right) = \log(x) - \log(y)$)

$= \log_{10}(10^S) + \log_{10}(T^2) - \log_{10}(A-B)$ (b/c $\log(x \cdot y) = \log(x) + \log(y)$)

$= S + 2\log_{10}(T) - \log_{10}(A-B)$ (b/c $\log_c(c^x) = x$ and $\log(a^b) = b\log(a)$)

(b) A scientist is growing two colonies of bacteria in separate petri dishes. Colony A is a strain that doubles its population every hour, and Colony B is a strain that triples its population every hour.
If the scientist starts growing Colony A at $t = 0$ hours, then its population is modeled by the function 2^t. If she starts growing Colony B four hours later, then its population is modeled by the function 3^{t-4}. At what time will the populations be equal? (Leave your answer in exact terms.)

We need to solve $2^t = 3^{t-4}$. There are many slightly different ways to do this that give answers that appear different, but equal the same number.

$2^t = 3^{t-4} \Rightarrow \log_3(2^t) = \log_3(3^{t-4}) \Rightarrow t\log_3(2) = t-4$

$\Rightarrow t\log_3(2) - t = -4 \Rightarrow t(\log_3(2) - 1) = -4$

$\Rightarrow t = \frac{-4}{\log_3(2) - 1}$

11. Compute the value of $\lim_{x\to 2} \dfrac{(x-2)\cos(x)}{x^2+x-6}$.

$$\frac{(x-2)\cos(x)}{x^2+x-6} = \frac{(x-2)\cos(x)}{(x-2)(x+3)} = \frac{\cos(x)}{x+3}$$

(when $x \neq 2$)

So, $$\lim_{x\to 2} \frac{(x-2)\cos(x)}{x^2+x-6} = \lim_{x\to 2} \frac{\cos(x)}{x+3} = \frac{\cos(2)}{5}$$

$\frac{\cos(x)}{x+3}$ continuous at $x=2$