MATH 2100 / 2105 / 2350 – HOMEWORK 6 (LAST ONE!!!)

due Wednesday, May 1, at the beginning of class

This homework assignment was written in LaTeX. You can find the source code on the course website.

Instructions: This assignment is due at the *beginning* of class. **Staple your work** together (do not just fold over the corner). Please write the questions in the correct order. If I cannot read your handwriting, you won't receive credit. Explain all reasoning.

1. Let $f : \mathcal{P}(\{1,2,3,4\}) \to \mathcal{P}(\{1,2,3\})$ be defined by $f(A) = A \setminus \{4\}$. Draw the arrow diagram for the function. Determine whether or not it's injective, surjective, and bijective. Make sure to justify your answers (either with the arrow diagram, or a formal proof).
2. Let $A = \{0,1,2,3\}$ and let $B = \{000, 001, 010, 011, 100, 101, 110, 111\}$ be the set of binary strings with three digits. Define $g : B \to A$ by $g(s) =$ [the number of 1s in s]. Draw the arrow diagram for the function. Determine whether or not it's injective, surjective, and bijective. Make sure to justify your answers (either with the arrow diagram, or a formal proof).
3. Let $c : \mathcal{P}(\{x,y,z\}) \to \mathcal{P}(\{x,y,z\})$ be the function with the rule $c(A) = \{x,y,z\} \setminus A$, and let $n : \mathcal{P}(\{x,y,z\}) \to \{0,1,2,3\}$ be the function such that $n(A)$ is the number of elements in the set A. Which composition makes sense, $c \circ n$ or $n \circ c$? For the one that is defined, give the domain, codomain, range, and draw the arrow diagram.
4. Prove that the function $h : \mathbb{N} \to \mathbb{N}$ defined by $h(n) =$ [the sum of the digits in n (in base 10)] is surjective. Prove that it's not injective.
5. Let $h : [2, \infty) \to (0, 1]$ be the function with the rule $h(x) = \dfrac{1}{x-1}$. Prove that h is a bijection by proving it is injective and surjective. Then compute $h^{-1}(x)$ and give its domain, codomain, and range.
6. Consider the set $S = \mathcal{P}(\{1,2,3,4\})$. Define $\Sigma(T)$ to be the sum of the elements in T. For example $\Sigma(\{1,3,4\}) = 8$. Define the relation $R = \{(A,B) \in S \times S : \Sigma(A) < \Sigma(B)\}$. Answer the following questions.
   (a) Is R reflexive?
   (b) Is R irreflexive?
   (c) Is R symmetric?
   (d) Is R antisymmetric?
   (e) Is R transitive?
   (f) Is R a partial order? If so, draw the Hasse diagram.
7. Let $S = [0, 4\pi)$ and define the relation $R = \{(a,b) \in S \times S : \sin(a) = \sin(b)\}$. Answer questions (a) – (f) from Question 6.
8. Let $A = \{1,4,7\}$. Give an example of a relation R on A that is
   (a) Transitive and reflexive but not antisymmetric.
   (b) Antisymmetric and reflexive but not transitive.
   (c) Antisymmetric and transitive but not reflexive.