MATH 2100 / 2105 / 2350 – HOMEWORK 5

due Wednesday, April 10, at the beginning of class

This homework assignment was written in LaTeX. You can find the source code on the course website.

Instructions: This assignment is due at the *beginning* of class. **Staple your work** together (do not just fold over the corner). Please write the questions in the correct order. If I cannot read your handwriting, you won't receive credit. Explain all reasoning.

1. Prove that any real number r that makes the equation $r - \dfrac{1}{r} = 5$ true must be irrational.
2. Prove that if $a + b + c + d \geq 26$, then either $a \geq 3$, $b \geq 7$, $c \geq 7$, or $d \geq 9$.
3. Use the pigeonhole principle to prove that given any five integers, there will be two that have a sum or difference divisible by 7.
4. Prove that at a completely full Milwaukee Bucks game at the new Fiserv Forum, there *must* be at least two people that have both the same birthday *and* the same first initial. (Note: you will have to look up the capacity of the new arena!)
5. Show that if you pick 17 points from a square with side length 4, then there must be 2 of those points that are within $\sqrt{2}$ of each other.
6. Prove or disprove: For any two sets A and B, $\mathcal{P}(A) \cup \mathcal{P}(B) = \mathcal{P}(A \cup B)$.
7. Prove or disprove: For any two sets A and B, $A \setminus B = A \cap \overline{B}$.
8. Prove the following set inequality:
$$(\{n^2 - 1 : n \in \mathbb{Z}\} \cap \{2k : k \in \mathbb{N}\}) \subseteq \{4m : m \in \mathbb{Z}\}.$$
9. Prove the following set inequality:
$$(\{6k + 1 : k \in \mathbb{Z}\} \cup \{6m - 1 : m \in \mathbb{Z}\}) \subseteq \{2n + 1 : n \in \mathbb{Z}\}.$$
10. Use induction to prove that for all $n \geq 1$, if A is a set of size n, then the number of subsets of A is 2^n. (In other words, $|\mathcal{P}(A)| = 2^{|A|}$.)