MATH 2100 / 2105 / 2350 – HOMEWORK 1

due Wednesday, **February 6**, at the beginning of class

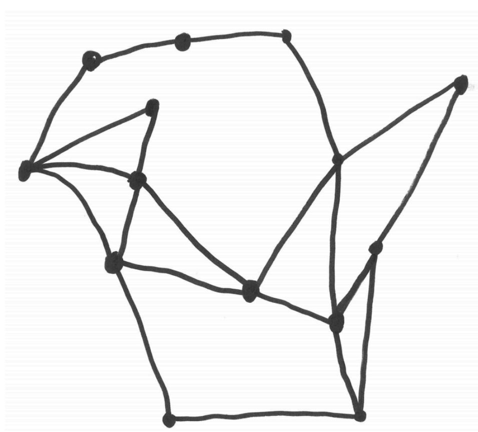
Sections 7.1, 1.3, 1.4, 1.5

This homework assignment was written in LaTeX. You can find the source code on the course website.

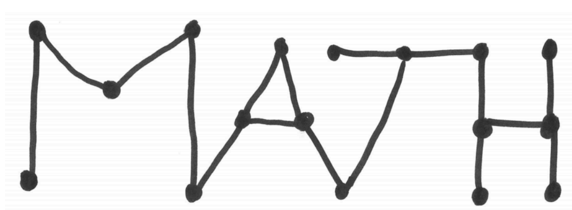
Instructions: This assignment is due at the *beginning* of class. **Staple your work** together (do not just fold over the corner). Please write the questions in the correct order. If I cannot read your handwriting, you won't receive credit. Explain all reasoning.

Mathematical Writing: An important component of this course is learning how to write mathematics correctly and concisely. Your goal should always be the convince the reader that you are correct! That means explaining your thinking and each step in your solution. We will talk more about this when we cover formal proofs in a few weeks, but for now I expect you to do the following: explain your reasoning, don't leave out steps, and use full sentences with correct spelling and grammar (including your use of math symbols). For example, don't write "$3 \in S \implies 3 \notin \overline{S}$"; instead, write "Since $3 \in S$, it follows that $3 \notin \overline{S}$".

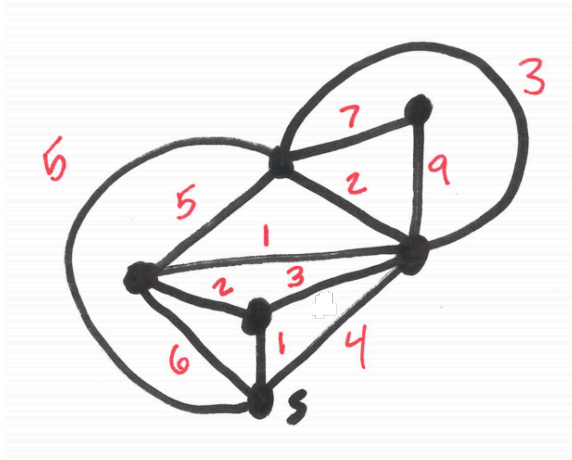
1. For the graph below, find an Eulerian path (if one exists), or explain why there can't be one (if one doesn't exist). Do the same thing for an Eulerian circuit.



2. For the graph below, find an Eulerian path (if one exists), or explain why there can't be one (if one doesn't exist). Do the same thing for an Eulerian circuit.



3. Apply Dijkstra's algorithm to the graph below to find the shortest distance between S and any other vertex. Draw a copy the graph with the appropriate d_v values at each step, indicating the current vertex.



4. You come across three inhabitants of an island: Alice, Bob, and Cindy. Alice says "Exactly two of us are lying." Bob says "I am telling the truth!" Cindy says "We're all liars."

   What are the possible combinations of whether each person is lying or telling the truth? (There could be none, one, or more than one.)

5. Give your own example, different from the ones in class, of a predicate $P(x, y)$ such that

$$\forall x, \exists y, P(x, y)$$

   and

$$\exists y, \forall x, P(x, y)$$

   mean different things. Explain what each version means. (Each student in the class should have a different answer.)

6. Write the negation of each of these sentences.

   (a) Every time you roll a "6", you have to take a card.
   (b) There is a day in your life better than every other day.
   (c) In every good book, there is a plot twist or surprise ending.
   (d) Every math course has a topic that everyone finds easy to do.

7. Let $Q(a, b) =$ "$a - b = 2ab$" and assume for the rest of this question that a and b are always integers (the integers are the set of positive and negative whole numbers (plus 0): $\{\ldots, -2, -1, 0, 1, 2, \ldots\}$). Which of the following are true? Justify your answers, stating explicitly whether you're justifying by giving a single example, or by stating something for all cases.

   (a) $Q(2, 1)$
   (b) $\exists x, Q(x, 0)$
   (c) $\forall x, Q(x, 0)$
   (d) $\exists y, Q(y, y)$
   (e) $\forall x, \exists y, Q(x, y)$
   (f) $\exists x, \forall y, Q(x, y)$

8. Form a predicate and a quantified statement that represents the following sentence: "There is a Marquette student who gets A's in all of her classes."

9. In each of the following pairs, one of the statements implies the other. Decide *without using a truth table* which one implies the other. Do this by thinking about the meaning of both sides, and, as always, explain your reasoning.

(a)	q	$p \to q$
(b)	$\forall y, \exists x, P(x,y)$	$\exists x, \forall y, P(x,y)$
(c)	$s \wedge (p \to q) \wedge ((\neg p) \to q)$	$(q \vee r)$
(d)	$(\forall x, P(x)) \vee (\forall x, \neg P(x))$	$(\exists x, P(x)) \to (\forall x, P(x))$

10. Express each of the following statements using predicates and quantifiers. (You do not need to prove them!)

(a) If n is a multiple of 5, then n ends in 5 or n ends in 0.

(b) If n is not a multiple of 3, then $n^2 - 1$ is a multiple of 3

(c) For all odd integers a and b, there is no real number x such that $x^2 + ax + b = 0$.

(d) For every real number y, if $y \geq 0$, then there exists $x \in \mathbb{R}$ such that $x^2 = y$.