MATH 2100 / 2105 / 2350 – EXAM 2

Wednesday, April 10

Name: ________________________________

Instructions: Please write your work neatly and clearly. **You must explain all reasoning. It is not sufficient to just write the correct answer.** You have 75 minutes to complete this exam. You may not use calculators, notes, or any other external resources.

Scores

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- To fully observe the rules governing exams and assignments regarding resource material, electronic aids, copying, collaborating with others, or engaging in any other behavior that subverts the purpose of the exam or assignment and the directions of the instructor.
- To turn in work done specifically for the paper or assignment, and not to borrow work either from other students, or from assignments for other courses.
- To complete individual assignments individually, and neither to accept nor give unauthorized help.
- To report any observed breaches of this honor code and academic honesty.

If you understand and agree to abide by this honor code, sign here:

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1. Determine if the following statement is true. If it is, prove it. If not, give a counterexample.

$$\text{For any two sets } A \text{ and } B,\ \mathcal{P}(A) \cup \mathcal{P}(B) = \mathcal{P}(A \cup B).$$

2. Prove that for all positive integers n,

$$\sum_{i=1}^{n}(3i-2) = \frac{n(3n-1)}{2}.$$

3. Suppose that every college basketball team plays 32 games in a season. (In reality, it's usually fewer.) Assuming that no games end in a tie, prove that there are at least three of the 68 teams in the March Madness tournament that have the exact same win-loss record.

4. Determine if the following statement is true. If it is, prove it. If not, give a counterexample.

$$\text{If } a \text{ is a rational number, then } a \cdot \sqrt{2} \text{ is irrational.}$$

5. Determine if the following statement is true. If it is, prove it. If not, give a counterexample.

If b divides a and c divides a, then bc divides a.

6. Determine if the following statement is true. If it is, prove it. If not, give a counterexample.

If N is not a multiple of 3, then $N^2 - 1$ is a multiple of 3.

7. Prove that if $p^2 + q^2 \neq 0$ then either $p \neq 0$ or $q \neq 0$.

8. Draw *both* the two-sided and one-sided arrow diagrams for the function $r : \mathcal{P}(\{1,2,3\}) \to \mathcal{P}(\{1,2,3\})$ defined by $r(S) = S \cup \{2\}$.

9. Prove that $9^n + 3$ is divisible by 4 for all natural numbers n.

10. Give an element-wise proof that

$$\{14k+4 : k \in \mathbb{N}\} \subseteq \{7\ell - 3 : \ell \in \mathbb{N}\} \cap \{2m : m \in \mathbb{N}\}.$$