MATH 60 – HOMEWORK 9

due Wednesday, May 30

Instructions: This assignment is due at the *beginning* of class. Staple your work together (do not just fold over the corner). Please write the questions in the correct order. If I cannot read your handwriting, you won't receive full credit. Every step of your answers must be fully justified to receive credit.

It is considered cheating and a violation of the Honor Code to look for answers to these problems on the internet.

- ◇ You may use a z-table on any question.
- ◇ Feel free to use a calculator for any basic arithmetic.
- ◇ You may use Wolfram Alpha to calculate the inverse of matrices or to solve systems of equations

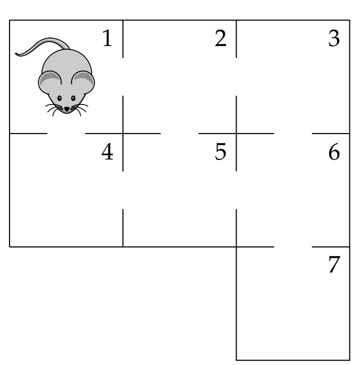
1. You are playing dodgeball with three other people, Elizabeth, Paige, and Henry. You always hit who you're aiming at with probability 1, Elizabeth hits who she's aiming at with probability 2/3, Paige hits who she's aiming at with probability 1/2, and Henry hits who he's aiming at with probability 1/3.

   You and the three others each pick up a ball at the same time, and each person throws their ball at the person (other than themselves) who has the highest accuracy. Anybody hit by a ball is out. (It's possible for two people to hit each other at the same time.) Then, everybody left picks up a ball all at the same time, and repeats. The game ends when only one person is left, or when no people are left.

   (a) Draw a Markov chain that represents all possible progressions of the game, where the states correspond to combinations of players that may be left in the game at any time.
   (b) Find the transition matrix.
   (c) What is the probability that the game eventually ends?
   (d) Find the probability that each person wins.
   (e) What is the probability that no one wins?
   (f) How many rounds do you expect the game to last?
   (g) What is the expected number of rounds in which only two players are left?

2. You flip a coin repeatedly, writing down 'H' each time it comes up heads and writing down 'T' each time it comes up tails. How many times do you expect to flip the coin until you write 'HTTH' consecutively? (Hint: Use a Markov Chain.)

3. A mouse starts in box 1 of the maze shown below. At each step, it moves into one of the adjoining rooms, each with equal probability.

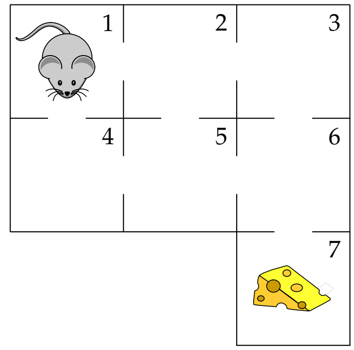


(a) Draw a Markov chain that represents the mouse's movements, and write down the transition matrix.

(b) Is this Markov chain absorbing? Is it ergodic? Is it regular? Justify your answers.

(c) In the long run, what proportion of time does the mouse spend in Room 7?

(d) Now suppose that a delicious piece of Havarti is placed in Room 7, as shown below.



Once the mouse enters Room 7, it stops to eat the cheese and never leaves. What is the expected number of steps that it will take for the mouse to reach Room 7?

(e) In the situation outlined in part (d), what is the expected number of times the mouse will be in Room 6 before it reaches the Cheese Paradise in Room 7?