MATH 60 – HOMEWORK 7

due Wednesday, May 16

Instructions: This assignment is due at the *beginning* of class. Staple your work together (do not just fold over the corner). Please write the questions in the correct order. If I cannot read your handwriting, you won't receive full credit. Every step of your answers must be fully justified to receive credit.

It is considered cheating and a violation of the Honor Code to look for answers to these problems on the internet.

You may use a z-table on any question.

1. What do Chebyshev's Inequality and the Law of Large Numbers say about the probability of getting at least 75 heads when flipping a fair coin 100 times? *Hint:* Improve your bound by using the fact that the binomial distribution is symmetric.

2. A share of common stock in the Pilsdorff beer company has price Y_n on the nth business day of the year. (Y_n is a random variable.) Finn observes that the price change $X_n = Y_{n+1} - Y_n$ appears to be a random variable with mean $\mu = 0$ and variance $\sigma^2 = 1/4$. If $Y_1 = 30$, find a lower bound for the following probabilities, under the assumption that the X_n's are mutually independent.

   (a) $P(25 \leq Y_2 \leq 35)$

   (b) $P(25 \leq Y_{11} \leq 35)$

   (c) $P(25 \leq Y_{101} \leq 35)$

3. Each student's score on a particular calculus final is a random variable with values in the range $[0, 100]$, mean 70, and variance 25.

   (a) Find the best lower bound you can (using only the tools we've learned), for the probability that a particular student's score will fall between 65 and 75.

   (b) If 100 students take the final and the students' grades are all mutually independent, find a lower bound for the probability that the class mean will fall between 65 and 75.

4. A fair coin is tossed 10,000 times.

   (a) Using a binomial distribution and Wolfram Alpha (or another similar tool) for experimentation, find the number m such that the probability of flipping between $5000 - m$ and $5000 + m$ heads is closest to $2/3$.

   (b) Answer the same question, but using only the Central Limit Theorem and a z-table.

5. Once upon a time, there were two railway trains competing for the passenger traffic of 1000 people leaving from Chicago at the same hour and going to Los Angeles. Assume that passengers are equally likely to choose each train and that their choices are mutually independent. How many seats must a train have to assure a probability of 0.99 or better of having a seat for each passenger? (Use the Central Limit Theorem.)

6. A club serves dinner to members only. They are seated at 12-seat tables. The manager observes over a long period of time that 95% of the time there are between six and nine full tables of members, and the remainder of the time the numbers are equally likely to fall above or below this range. Assume that each member decides to come with a given probability p and that the decisions are mutually independent. How many members are in the club?