MATH 60 – HOMEWORK 6

due Wednesday, May 9

Instructions: This assignment is due at the *beginning* of class. Staple your work together (do not just fold over the corner). Please write the questions in the correct order. If I cannot read your handwriting, you won't receive full credit. Every step of your answers must be fully justified to receive credit.

It is considered cheating and a violation of the Honor Code to look for answers to these problems on the internet.

---

1. Let $X_1, X_2, \ldots, X_k$ be k random variables that are mutually independent and uniformly distributed on the interval $[0, 1]$. Define a new random variable $Y = \min(X_1, X_2, \ldots, X_k)$ such that the value of Y is the smallest of the values of $X_1, X_2, \ldots, X_k$. Find $f_Y(y)$, $F_Y(y)$, $\mathbb{E}[Y]$, and $\mathrm{Var}(Y)$. Do not use WolframAlpha to compute integrals. *Hint:* Try u-substitution.

2. A particular molecule emits radiation at an average rate of 1 particle every 90 seconds. Suppose that a detection device is switched on at time $t = 0$. Let T_4 be the time in minutes at which the fourth burst of radiation is emitted. Find $\mathbb{P}(4 \leq T_4 \leq 7)$.

3. Prove that if X has an exponential distribution with rate λ then the random variable $Z = \lceil X \rceil$ defined by

$$Z(\omega) = [\text{the smallest integer larger than or equal to } X(\omega)] = [X(\omega), \text{rounded up to the nearest integer}]$$

   has a geometric distribution on the nonnegative integers.

4. Transistors produced by one machine have a lifetime that is exponentially distributed with mean 100 hours. Those produced by a second machine have an exponentially distributed lifetime with mean 200 hours. A package of 12 transistors contains 4 produced by the first machine and 8 produced by the second. Let X be the lifetime of a transistor picked at random from this package.

   (a) Find the pdf $f_X(x)$ and the cdf $F_X(x)$.

   (b) Calculate $P(X \geq 200)$.

   (c) Find $\mathbb{E}[X]$ and $\mathrm{Var}(X)$.

5. Let X be a random variable with uniform distribution over the real interval $[\ell, r]$. For what values of a and b is the random variable $Y = aX + b$ uniformly distributed over the real interval $[0, 1]$?

6. A school wishes to accept 2000 students for their freshman class, and they expect 20,000 applications. In order to make their admissions decisions very easy, the only criterion they will use is SAT score. So, their goal is to accept a student if and only if their SAT score is in the top 10%. However, because their computer system is so old, the applications only come in one at a time, and they must decide whether to accept or reject before moving on to the next application. Assuming that SAT scores are normally distributed with a mean of 1000 and a standard deviation of 200, how should they set the score threshold to end up with as close to 2000 students as possible? Give your answer first symbolically (in terms of a pdf, cdf, etc), then use a normal distribution table[1] to provide a numerical answer.

7. Suppose that the height, in inches, of a 25-year old man is a normal random variable with parameters $\mu = 71$ and $\sigma^2 = 6.25$. What percentage of 25-year old men are over 6 feet 2 inches tall? What percentage of men over 6 feet tall are over 6 feet 5 inches?

---

[1] For example: http://www.stat.ufl.edu/~athienit/Tables/Ztable.pdf. The numbers in the leftmost column represent the number of standard deviations from the mean up to 1 decimal place, and the numbers along to top row then refine to the second decimal place. Please ask me if you have questions about this.