MATH 60 – HOMEWORK 5

due Wednesday, May 2

Instructions: This assignment is due at the *beginning* of class. Staple your work together (do not just fold over the corner). Please write the questions in the correct order. If I cannot read your handwriting, you won't receive full credit. Every step of your answers must be fully justified to receive credit.

It is considered cheating and a violation of the Honor Code to look for answers to these problems on the internet.

1. Assume S and T are independent discrete random variables both on the sample space $\mathbb{N}$ and $Q = S + T$. Find (and prove) an expression for the probability distribution function of Q (i.e., $\mathbb{P}(Q = k)$) for an arbitrary nonnegative integer k in terms of the probability distribution functions for S and T.

2. Let X and Y be independent binomial random variables, such that both X and Y have the same probability of success p and have number of trials N_X and N_Y.
   (a) Describe the random variable $Z = X + Y$, giving its probability distribution function, expected value, variance, and standard deviation.
   (b) Repeat part (a) for $W = X - Y$.

3. Let $P_1, P_2, \ldots$ be iid Poisson random variables with rate λ.
   (a) Prove that $P_1 + P_2$ is itself a Poisson random variable with rate 2λ.
   (b) Prove that $P_1 + P_2 + \cdots + P_n$ is itself a Poisson random variable with rate $n\lambda$. You may use the fact that
   $$\sum_{k=0}^{n} A^{n-k} \binom{n}{k} = (A+1)^n.$$
   (c) Give an intuitive explanation for why (a) and (b) make sense.

4. An airline finds that 4% of the passengers that make reservations on a particular flight will not show up. Consequently, their policy is to sell 100 reserved seats on a plane that has only 98 seats. Find the probability that every person who shows up will get a seat, using
   (a) a binomial distribution, then Wolfram Alpha or some other tool to get a numerical answer
   (b) a Poisson approximation, then a calculator or Wolfram Alpha to get a numerical answer

5. A baker blends 600 raisins and 400 chocolate chips into a dough mix and, from this, makes 500 cookies. Answer the following questions with both binomial and Poisson distributions, as in the previous question.
   (a) Find the probability that a randomly picked cookie will have no raisins.
   (b) Find the probability that a randomly picked cookie will have exactly two chocolate chips.
   (c) Find the probability that a randomly picked cookie will have at least two bits (raisins or chips) in it.

6. One way to get out of jail in Monopoly is to roll doubles (two-of-a-kind) on a pair of fair six-sided dice. You get one attempt to roll doubles on each turn. Let X be the number of turns required to get out of jail. Ignoring the Monopoly rule that says you have to pay to leave jail if you fail three times, find $\mathbb{P}(X = k)$, $\mathbb{E}[X]$, and $\mathrm{Var}(X)$.