# MATH 60 – HOMEWORK 4

## due Wednesday, April 25

**Instructions:** This assignment is due at the *beginning* of class. Staple your work together (do not just fold over the corner). Please write the questions in the correct order. If I cannot read your handwriting, you won't receive full credit. Every step of your answers must be fully justified to receive credit.

**It is considered cheating and a violation of the Honor Code to look for answers to these problems on the internet.**

1. (a) Suppose that $Z$ is a random variable that takes only three distinct values: $a$, $b$, and $c$, i.e., $P(Z = z)$ is nonzero only when $z \in \{a, b, c\}$. Prove that $Z$ is *completely determined* by $\mathbb{E}[Z]$ and $\mathbb{E}[Z^2]$. In other words, prove that if all you know is $\mathbb{E}[Z]$ and $\mathbb{E}[Z^2]$, then this is enough information to recover $\mathbb{P}(Z = a)$, $\mathbb{P}(Z = b)$, and $\mathbb{P}(Z = c)$.

   (b) Show that the statement in Question 1 is no longer true for a random variable that takes four possible values $a$, $b$, $c$, and $d$, i.e., that such a random variable is not completely determined by $\mathbb{E}[Z]$ and $\mathbb{E}[Z^2]$.

2. Consider flipping a fair coin. Suppose that you flip the coin repeatedly until *you get a heads then a tails consecutively, in that order*. For example, some flipping sequences are: HT, HHHHHT, and TTTTHHHT. Let $X$ be the random variable for the number of flips completed. Find $\mathbb{E}[X]$ and $\text{Var}(X)$. You may use Wolfram Alpha or another tool to compute the value of any infinite sums. **See note below.**

3. Suppose you play the following game. A randomly chosen permutation of the numbers $\{1, 2, \ldots, n\}$ is chosen. You get a dollar for the first entry in the permutation, regardless of what it is, and then reading left-to-right you get another dollar for each entry that is larger than all of the entries that came before. For example, if $n = 9$ and the randomly chosen permutation is $3, 1, 6, 4, 2, 8, 7, 9, 5$ then you'd win \$4: one for the 3, one for the 6, one for the 8, and one for the 9. For which values of $n$ is this game profitable if it costs \$4 to play? For which is it profitable if it costs \$10 to play? Again, you may use Wolfram Alpha or another tool to compute any large summations. **See note below.**

4. A couple decides to have children until either they have a girl, or until they have three children total, whichever comes first. Find the expected value, variance, and standard deviation of both the number of boys and the number of girls that this family will end up with.

5. The average household income in Newport, NH is about \$50,000.

   (a) Compute an upper bound for the proportion of households with incomes over \$150,000.

   (b) Now, compute an upper bound for the same thing assuming that the standard deviation in household income is \$30,000.

**Note:** If you have any questions about how to compute summations with Wolfram Alpha, or another tool, please ask me! Also, make sure to state in your write-up something like "I used Wolfram Alpha to compute that this summation is equal to…"