MATH 60 – HOMEWORK 1

due Wednesday, April 4

Instructions: This assignment is due at the *beginning* of class. Staple your work together (do not just fold over the corner). Please write the questions in the correct order. If I cannot read your handwriting, you won't receive full credit.

Every step of your answers must be fully justified to receive credit.

1. Suppose that you have an unlimited number of red ping pong balls and yellow ping pong balls and that you have 10 distinguishable buckets labeled 1 through 10. How many ways are there to distribute n ping pong balls into the ten buckets according to the following restrictions:

   ◇ each even numbered bucket has at least one red ping pong ball and at least two yellow ping pong balls,

   ◇ each odd numbered bucket has at least three red ping pong balls?

   Your answer should have no summation signs.

2. A group of forty Math 60 students goes to the movies, and they all sit in one long row of forty chairs. There are two groups of three best friends: Alice, Bob, and Cindy are all best friends, and Xavier, Yolanda, and Zeke are all best friends. If all forty students are assigned seats at random (so all seating arrangements are equally likely), then what is the probability that each group of three best friends is sitting in three consecutive seats (in any order)? Your answer should have no summation signs.

3. A tennis intramural team has $4n$ players. A *doubles match* consists of two teams of two people each. How many ways can you group the players into doubles matches where each player is in exactly one doubles match? How many ways can we do it if we also choose who serves first on each team? (To clarify, if teams (A, B) and (C, D) are playing a doubles match together, pick which of A or B serves first on their turn and which of C or D serves first on their turn. Don't pick which team serves first in each match.) Your answer should have no summation signs.

4. Given integers x and y, we say that x *divides* y if there is some other integers k such that $x \cdot k = y$. For example, 5 divides 10 but 4 does not divide 10.

   Prove that if x and y are integers, if x divides y, and if x is even, then y is even.

5. Prove *without using the formula for binomial coefficients* that

$$\binom{m+n}{2} = \binom{m}{2} + \binom{n}{2} + mn.$$

   Hint: Show that the quantity on the left-hand side and the quantity on the right-hand side count the same thing.

6. Let A, B, and C be sets. Prove that

$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C).$$

   Hint: The best way to show that two sets S and T are equal is in two separate steps. First prove $S \subseteq T$, then prove $T \subseteq S$.

7. Let A and B be events in a sample space Ω of equally likely outcomes. Prove the following:

   (a) If $A \subseteq B$ then $\mathbb{P}(A) \leq \mathbb{P}(B)$.

   (b) $\mathbb{P}(\overline{A}) = 1 - \mathbb{P}(A)$.

   (c) If $A \cap B = \emptyset$ then $\mathbb{P}(A \cup B) = \mathbb{P}(A) + \mathbb{P}(B)$.

   (d) $\mathbb{P}(A \cup B) = \mathbb{P}(A) + \mathbb{P}(B) - \mathbb{P}(A \cap B)$. *Hint:* $A \cup B = A \cup (B \setminus A)$