

Statement	Ways to Prove it	Ways to Use it	How to Negate it
$p$	<ul style="list-style-type: none"> <li>• Prove that <math>p</math> is true.</li> <li>• Assume <math>p</math> is false, and derive a contradiction.</li> </ul>	<ul style="list-style-type: none"> <li>• <math>p</math> is true.</li> <li>• If <math>p</math> is false, you have a contradiction.</li> </ul>	not $p$
$p$ and $q$	<ul style="list-style-type: none"> <li>• Prove <math>p</math>, and then prove <math>q</math>.</li> </ul>	<ul style="list-style-type: none"> <li>• <math>p</math> is true.</li> <li>• <math>q</math> is true.</li> </ul>	(not $p$ ) or (not $q$ )
$p$ or $q$	<ul style="list-style-type: none"> <li>• Assume <math>p</math> is false, and deduce that <math>q</math> is true.</li> <li>• Assume <math>q</math> is false, and deduce that <math>p</math> is true.</li> <li>• Prove that <math>p</math> is true.</li> <li>• Prove that <math>q</math> is true.</li> </ul>	<ul style="list-style-type: none"> <li>• If <math>p \Rightarrow r</math> and <math>q \Rightarrow r</math> then <math>r</math> is true.</li> <li>• If <math>p</math> is false, then <math>q</math> is true.</li> <li>• If <math>q</math> is false, then <math>p</math> is true.</li> </ul>	(not $p$ ) and (not $q$ )
$p \Rightarrow q$	<ul style="list-style-type: none"> <li>• Assume <math>p</math> is true, and deduce that <math>q</math> is true.</li> <li>• Assume <math>q</math> is false, and deduce that <math>p</math> is false.</li> </ul>	<ul style="list-style-type: none"> <li>• If <math>p</math> is true, then <math>q</math> is true.</li> <li>• If <math>q</math> is false, then <math>p</math> is false.</li> </ul>	$p$ and (not $q$ )
$p \iff q$	<ul style="list-style-type: none"> <li>• Prove <math>p \Rightarrow q</math>, and then prove <math>q \Rightarrow p</math>.</li> <li>• Prove <math>p</math> and <math>q</math>.</li> <li>• Prove (not <math>p</math>) and (not <math>q</math>).</li> </ul>	<ul style="list-style-type: none"> <li>• Statements <math>p</math> and <math>q</math> are interchangeable.</li> </ul>	( $p$ and (not $q$ )) or ((not $p$ ) and $q$ )
$(\exists x \in S) P(x)$	<ul style="list-style-type: none"> <li>• Find an <math>x</math> in <math>S</math> for which <math>P(x)</math> is true.</li> </ul>	<ul style="list-style-type: none"> <li>• Say “let <math>x</math> be an element of <math>S</math> such that <math>P(x)</math> is true.”</li> </ul>	$(\forall x \in S) \text{ not } P(x)$
$(\forall x \in S) P(x)$	<ul style="list-style-type: none"> <li>• Say “let <math>x</math> be any element of <math>S</math>.” Prove that <math>P(x)</math> is true.</li> </ul>	<ul style="list-style-type: none"> <li>• If <math>x \in S</math>, then <math>P(x)</math> is true.</li> <li>• If <math>P(x)</math> is false, then <math>x \notin S</math>.</li> </ul>	$(\exists x \in S) \text{ not } P(x)$

Table 1: Logic in a nutshell.