MATH 118 – HOMEWORK 2

Spring 2017

assigned Wednesday, April 19
due Wednesday, May 3

Each question is worth 10 points. The maximum possible score on this assignment is 50 points.

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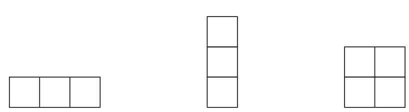
1. Let $\mathcal{X}$ and $\mathcal{Y}$ be regular languages, and consider the language
$$\mathcal{L} = \{x \in \mathcal{X} : \exists y \in \mathcal{Y} \text{ such that } |x| = |y|\}.$$
Use homomorphisms to prove that $\mathcal{L}$ is regular.

2. Let $\mathcal{L}$ be a regular language and define
$$\textsc{Irred}(\mathcal{L}) = \{w \in \mathcal{L} : xy \neq w \text{ for all } x, y \in \mathcal{L}\}.$$
Prove that $\textsc{Irred}(\mathcal{L})$ is regular.

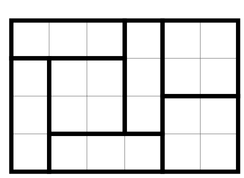
3. Suppose that $f : \mathcal{P}(\Sigma^*) \to \mathcal{P}(\Delta^*)$ and $g : \mathcal{P}(\Delta^*) \to \mathcal{P}(\Gamma^*)$ are transductions. Prove that $f \circ g$ is a transduction.
(*Hint:* It may help to first prove that any transducer T can be converted to an equivalent transducer $\widehat{T}$ whose transitions are labeled x/y for *single letters or empty words* x and y rather than whole words.)

4. Let M be a DFA that accepts the language
$$\mathcal{L}_n = \{w \in \{\texttt{0}, \texttt{1}\}^* : \text{the } n\text{th digit from the right is } \texttt{1}\}.$$
Prove that M must have at least 2^n states.

5. Consider the set of tilings of a $4 \times n$ rectangular strip using the tiles



Let $f(x, y)$ be the generating function for these tilings where x tracks the width of the rectangle (n) and y tracks the number of tiles used. Find $f(x, y)$.
For example, the tiling



contributes to the monomial x^6y^7.
