

MATH 118 – HOMEWORK 1

Spring 2017

assigned Wednesday, April 5
due Wednesday, April 19

Each question is worth 10 points. The maximum possible score on this assignment is 50 points.

1. Give a DFA that accepts the language of words over $\{0, 1\}$ that either have even length and end with 0 or have odd length and end with 1.
2. Give a regular expression that matches the language in Exercise 1.
3. Prove that for any n , the language

$$\text{BINDIV}_n = \{w \in \{0, 1\}^* : w \text{ is a valid binary number that is divisible by } n\}.$$

By *valid binary number*, we mean w has length at least 1 and must start with a 1 (unless $w = 0$). For example, $1111 \in \text{BINDIV}_5$, but $01111 \notin \text{BINDIV}_5$.

Submit AT MOST one of the following two questions.

4. Prove that if \mathcal{L} is recognizable then so is

$$\text{HALFPALINDROME}(\mathcal{L}) = \{w : ww^R \in \mathcal{L}\}$$

5. Prove that if \mathcal{L} is recognizable then so is

$$\text{HALFDOUBLE}(\mathcal{L}) = \{w : ww \in \mathcal{L}\}$$

Submit AT MOST one of the following two questions.

6. Prove that if \mathcal{L} is recognizable then so is

$$\text{INFPOW}(\mathcal{L}) = \{w : w^n \in \mathcal{L} \text{ for infinitely many } n \geq 0\}$$

(Hint: Question 5 might be a good warmup for this problem.)

7. Prove that if \mathcal{L} is recognizable then so is

$$\text{EXP}(\mathcal{L}) = \{x : \text{there exists } y \in \Sigma^* \text{ such that } xy \in \mathcal{L} \text{ and } |y| = 2^{|x|}\}$$

The set of answers you submit should be an element of the set $\{1\} \times \{2\} \times \{3\} \times \{4, 5\} \times \{6, 7\}$.