

Math 22 Spring 2016, Homework 9, will not be collected

Instructions: Write your answers neatly and clearly on straight-edged paper, use complete sentences, and label any diagrams. List problems in numerical order and staple all pages together. Start each problem on a new page. Please show your work; no credit is given for solutions without work or justification. If you are not sure what you are allowed to assume for a problem, ask!

1. Produce an orthonormal basis for the subspace W of \mathbb{R}^4 with the basis

$$\left\{ \begin{bmatrix} 1 \\ -2 \\ 0 \\ 5 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \\ 3 \end{bmatrix} \right\}.$$

2. Let $A = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}$. Write A as QR where the columns of Q form an orthonormal basis for $\text{Col}(A)$ and R is upper triangular with positive entries on its diagonal.
3. Answer the following questions about the page rank algorithm.
 - (a) State the definition of a *probability vector*.
 - (b) State the definition of a *stochastic matrix*.
 - (c) State the definition of a *steady-state vector*.
 - (d) Explain how each of the above definitions relates to the page rank algorithm.
4. Answer the following questions about the jpeg algorithm.
 - (a) What is the goal of the jpeg algorithm?
 - (b) Why is a change-of-basis employed?
 - (c) Explain what the Discrete Cosine Transformation is, and why it's useful.
 - (d) Explain the purpose of the quantization step.
 - (e) When a jpeg is over-compressed, you get an effect called *blocking* or *artifacting*, where square-shaped distortions are visible, as in the picture below.



Can you figure out why this happens? (In particular, why does it create this visible checkerboard distortion?)