Math 22 Spring 2016, Homework 8, due Wednesday, May 25

Instructions: Write your answers neatly and clearly on straight-edged paper, use complete sentences, and label any diagrams. List problems in numerical order and staple all pages together. Start each problem on a new page. Please show your work; no credit is given for solutions without work or justification. If you are not sure what you are allowed to assume for a problem, ask!

Total: 20 points

1.) (6 points) a.) Find the value of k such that the vectors

$$\begin{bmatrix} 2 \\ -3 \\ k \end{bmatrix} \text{ and } \begin{bmatrix} -2 \\ 5 \\ -1 \end{bmatrix}$$

are orthogonal in $\mathbb{R}^3$.

b.) Let W be the subspace of $\mathbb{R}^3$ given by $W = \text{Span}\{\mathbf{u}, \mathbf{v}\}$ where

$$\mathbf{u} = \begin{bmatrix} -3 \\ 4 \\ 1 \end{bmatrix} \text{ and } \mathbf{v} = \begin{bmatrix} -2 \\ 5 \\ 1 \end{bmatrix}.$$

Find a basis for the orthogonal complement $W^{\perp}$ of W. What geometric object is $W^{\perp}$?

2.) (5 points) Does the inner product in $\mathbb{R}^n$ satisfy the following identity for all vectors $\mathbf{u}$ and $\mathbf{v}$?

$$\mathbf{u} \bullet \mathbf{v} = \frac{1}{4}\Big(\|\mathbf{u} + \mathbf{v}\|^2 - \|\mathbf{u} - \mathbf{v}\|^2\Big)$$

If it does, give a derivation of the formula. If not, give two vectors $\mathbf{u}$ and $\mathbf{v}$ (in $\mathbb{R}^3$) for which it fails to be true.

3.) (9 points) a.) Verify that the following set of vectors in $\mathbb{R}^4$ is an orthogonal set:

$$\mathbf{u}_1 = \begin{bmatrix} 1/2 \\ 1 \\ 1/2 \\ 1/2 \end{bmatrix}, \quad \mathbf{u}_2 = \begin{bmatrix} 1 \\ 1 \\ -2 \\ -1 \end{bmatrix}, \quad \mathbf{u}_3 = \begin{bmatrix} -1/\sqrt{7} \\ 1/\sqrt{7} \\ 1/\sqrt{7} \\ -2/\sqrt{7} \end{bmatrix}.$$

b.) Let $W = \text{Span}\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3\}$. Find a matrix P such that the orthogonal projection onto the subspace W is given by

$$\text{proj}_W \mathbf{y} = P\,\mathbf{y}.$$