Math 22 Spring 2016, Homework 7, due Wednesday, May 18

Instructions: Write your answers neatly and clearly on straight-edged paper, use complete sentences, and label any diagrams. List problems in numerical order and staple all pages together. Start each problem on a new page. Please show your work; no credit is given for solutions without work or justification. If you are not sure what you are allowed to assume for a problem, ask!

1. *(4 points)* Consider the following linear transformations (see Example 3 and Tables 1 & 3 on pages 72 - 74 of the book).
   Determine whether the corresponding standard matrix of the given transformation has a real eigenvalue. If so, state **one** eigenvalue and a corresponding eigenvector. Justify your answer.
   (a) $T : \mathbb{R}^2 \to \mathbb{R}^2, v \mapsto T(v) = 3v$.
   (b) $S : \mathbb{R}^2 \to \mathbb{R}^2$, where S is a reflection with respect to the line $\{c \cdot \begin{bmatrix} -1 \\ 1 \end{bmatrix}, \quad \text{where} \quad c \in \mathbb{R}\}$.
   (c) A horizontal shear in $\mathbb{R}^2$.
   (d) $R(\frac{\pi}{2}) : \mathbb{R}^2 \to \mathbb{R}^2$, where $R(\frac{\pi}{2})$ is the counterclockwise rotation about the origin with angle $\frac{\pi}{2}$.

2. *(6 points)* Let A be the 4×4 matrix and $\{v_1, v_2, v_3, v_4, v_5\}$ the vectors in $\mathbb{R}^4$ below:
$$A = \begin{bmatrix} 5 & -4 & -2 & 4 \\ 3 & -2 & 0 & 2 \\ 0 & 0 & 2 & 2 \\ 0 & 0 & 2 & 2 \end{bmatrix} \quad \text{and} \quad v_1 = \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix}, v_2 = \begin{bmatrix} 2 \\ 2 \\ 3 \\ 3 \end{bmatrix}, v_3 = \begin{bmatrix} 4 \\ 3 \\ 0 \\ 0 \end{bmatrix}, v_4 = \begin{bmatrix} 2 \\ 4 \\ -1 \\ 1 \end{bmatrix}, v_5 = \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix}.$$
   (a) Determine which of these vectors are eigenvectors of A. For the ones that are eigenvectors state the corresponding eigenvalue.
   (b) Find a basis of $\mathbb{R}^4$ consisting of eigenvectors of A.

3. *(4 points)* Find the characteristic polynomial of the following matrices, then determine the eigenvalues and their multiplicity. Justify your answer.
$$A = \begin{bmatrix} 3 & 2 \\ 1 & 2 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 3 & 2 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}, \quad C = \begin{bmatrix} -3 & 2 & 1 \\ 0 & 2 & 0 \\ 4 & 2 & -1 \end{bmatrix}, \quad D = \begin{bmatrix} -2 & -4 & -2 & 4 \\ 0 & -2 & 9 & 2 \\ 0 & 0 & 6 & 2 \\ 0 & 0 & -\frac{1}{2} & 4 \end{bmatrix}.$$

4. *(6 points)* Let A be the 2×2 matrix $A = \begin{bmatrix} 3 & 5 \\ \frac{9}{2} & \frac{5}{2} \end{bmatrix}$.
   (a) Find the eigenvalues of A.
   (b) Find a basis of $\mathbb{R}^2$ consisting of eigenvectors of A.
   (c) Diagonalize the matrix A.
   (d) Calculate A^7.
   (e) Diagonalize the inverse matrix A^{-1} of A.

   Hint: For (d) and (e) you can use the information from (c).