

Math 22 Spring 2016, Homework 6, due Tuesday, May 10

Instructions: Write your answers neatly and clearly on straight-edged paper, use complete sentences, and label any diagrams. List problems in numerical order and staple all pages together. Start each problem on a new page. Please show your work; no credit is given for solutions without work or justification. If you are not sure what you are allowed to assume for a problem, ask!

1. (6 points)

(a) Suppose you have a matrix A with 4 rows and n columns, and the columns of A span \mathbb{R}^4 . What is the dimension of $\text{Nul } A$? What does this tell you about n ?

(b) Suppose you have a matrix B with m rows and 5 columns, and $\dim(\text{Nul } B) = 2$. What is the rank of B ? What does this tell you about m ?

2. (6 points) Let $A = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 1 & 2 & 4 & 5 \\ 3 & 6 & 0 & 3 \end{pmatrix}$.

Find bases for $\text{Col } A$, $\text{Row } A$, and $\text{Nul } A$. What are the dimensions of these subspaces?

3. (8 points) Let $\mathcal{B} = \left\{ \begin{bmatrix} 3 \\ 5 \end{bmatrix}, \begin{bmatrix} 2 \\ 3 \end{bmatrix} \right\}$ and $\mathcal{C} = \left\{ \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \end{bmatrix} \right\}$ be two bases for \mathbb{R}^2 .

(a) Find the change-of-coordinates matrix from \mathcal{B} to the standard basis.

(b) Find the change-of-coordinates matrix from the standard basis to \mathcal{C} .

(c) Use your answers from (a) and (b) to find the change-of-coordinates matrix from \mathcal{B} to \mathcal{C} .

(d) Suppose $[\mathbf{x}]_{\mathcal{B}} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$. Find \mathbf{x} , and $[\mathbf{x}]_{\mathcal{C}}$.