Math 22 Spring 2016, Homework 5, due Wednesday, May 4

Instructions: Write your answers neatly and clearly on straight-edged paper, use complete sentences, and label any diagrams. List problems in numerical order and staple all pages together. Start each problem on a new page. Please show your work; no credit is given for solutions without work or justification. If you are not sure what you are allowed to assume for a problem, ask!

1. *(6 points)* Let $A = \begin{bmatrix} 2 & 0 & 6 & 2 & 2 \\ 6 & 2 & 14 & 14 & 2 \\ 4 & 2 & 8 & 12 & 2 \end{bmatrix}$.

   (a) The null space of A is a subspace of $\mathbb{R}^d$ for what value of d?

   (b) The column space of A is a subspace of $\mathbb{R}^d$ for what value of d?

   (c) Find a basis for the null space of A.

   (d) Find a basis for the column space of A.

2. *(6 points)* The set of upper triangular 3×3 matrices, i.e., those of the form

$$\begin{bmatrix} a & b & c \\ 0 & d & e \\ 0 & 0 & f \end{bmatrix}$$

   for $a, b, c, d, e, f \in \mathbb{R}$, form a vector space V.

   (a) Verify that the set

$$\mathcal{B} = \left\{ \begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 & -1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \right\}$$

   is a basis for V.

   (b) Using this basis $\mathcal{B}$, calculate $[\,M\,]_{\mathcal{B}}$, where

$$M = \begin{bmatrix} 3 & 0 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix}.$$

3. *(4 points)* Find a basis for the space spanned by the vectors

$$\left\{ \begin{bmatrix} 2 \\ 3 \\ 10 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ -2 \\ -3 \\ 1 \end{bmatrix}, \begin{bmatrix} 3 \\ 1 \\ 5 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 2 \\ 7 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 2 \\ 1 \end{bmatrix} \right\}.$$

4. *(4 points)* Let $\boldsymbol{p}_1(t) = 1 - t$, $\boldsymbol{p}_2(t) = 1 - t^2$, and $\boldsymbol{p}_3(t) = 1 + t + t^2$. Let $\mathcal{B} = \{\boldsymbol{p}_1, \boldsymbol{p}_2, \boldsymbol{p}_3\}$.

   (a) Use coordinate vectors to show that $\mathcal{B}$ is a basis for $\mathbb{P}_2$.

   (b) Suppose $\boldsymbol{q}(t)$ has the property that $[\,\boldsymbol{q}\,]_{\mathcal{B}} = \begin{bmatrix} 2 \\ 1 \\ -2 \end{bmatrix}$. Find $\boldsymbol{q}(t)$.