

**Math 22 Spring 2016, Homework 4, due Wednesday, April 27**

**Instructions:** Write your answers neatly and clearly on straight-edged paper, use complete sentences, and label any diagrams. List problems in numerical order and staple all pages together. Start each problem on a new page. Please show your work; no credit is given for solutions without work or justification. If you are not sure what you are allowed to assume for a problem, ask!

**Total:** 20 points

1.) (4 points) Consider the linear transformation  $T : \mathbb{R}^4 \rightarrow \mathbb{R}^4$  given by

$$T(x_1, x_2, x_3, x_4) = (3x_1 + 4x_2 + 7x_3 + 4x_4, x_2 + 4x_3 + 6x_4, 2x_3 + 8x_4, x_4).$$

a.) Find the standard matrix  $A$  of  $T$ .

b.) Show that the transformation  $T$  is invertible.

c.) Find a formula for the inverse transformation  $T^{-1} : \mathbb{R}^4 \rightarrow \mathbb{R}^4$ .

2.) (3 points) Compute the determinant of the following matrix (using the most computationally efficient method!)

$$A = \begin{pmatrix} 6 & 2 & 5 & 4 \\ 0 & 0 & -2 & 0 \\ 1 & 1 & 0 & 0 \\ 7 & 3 & 6 & 0 \end{pmatrix}$$

3.) (5 points) a.) Let  $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$  and  $B = \begin{pmatrix} e & f \\ g & h \end{pmatrix}$ . Compute  $AB$  and use this to show that  $\det(AB) = \det(A)\det(B)$ .

b.) Let  $A$  be as in part (a). Let  $I_2$  be the  $2 \times 2$  identity matrix. Show that

$$\det(A + I_2) = \det(A) + \det(I_2) \text{ if and only if } a + d = 0.$$

4.) (8 points) a.) Consider the set  $V = \mathbb{R}^2$  with the usual vector addition but a NEW “scalar multiplication” operation defined by the formula:

$$\text{If } \mathbf{u} = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} \text{ and } c \text{ is a scalar then } c\mathbf{u} =_{\text{def}} \begin{bmatrix} cu_1 \\ 0 \end{bmatrix}.$$

Which vector space axioms (on page 190) are satisfied or not satisfied? Explain.

b.) Consider the set  $P_\pi$  of all polynomials in  $\mathbb{P}_n$  such that  $p(\pi) = 0$ . Is  $P_\pi$  a subspace of  $\mathbb{P}_n$ ? Why or why not? (See Example 4, page 192.)