Math 22 Spring 2016, Homework 2, due Wednesday, April 13

Instructions: Write your answers neatly and clearly on straight-edged paper, use complete sentences, and label any diagrams. List problems in numerical order and staple all pages together. Start each problem on a new page. Please show your work; no credit is given for solutions without work or justification. If you are not sure what you are allowed to assume for a problem, ask!

1. *(5 points)* (a) Describe the solution set of the following homogeneous system in parametric vector form.

$$\begin{array}{rcrcrcl} x_1 & + & 2x_2 & - & 3x_3 & = & 0 \\ 2x_1 & + & x_2 & - & 3x_3 & = & 0 \\ -x_1 & + & x_2 & & & = & 0\,. \end{array}$$

   (b) Is $\mathbf{v} = \begin{pmatrix} -3 \\ -3 \\ 0 \end{pmatrix}$ a linear combination of $\left\{ \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}, \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix} \right\}$? Use your answer from part (a).

2. *(3 points)* (a) Let $\mathbf{v} = \begin{pmatrix} 1 \\ -2 \\ 3 \end{pmatrix}$, $\mathbf{w} = \begin{pmatrix} 0 \\ 1 \\ -2 \end{pmatrix}$, and $\mathbf{z} = \begin{pmatrix} 2 \\ -3 \\ 4 \end{pmatrix}$. It can be shown that $2\mathbf{v} + \mathbf{w} = \mathbf{z}$. Use this fact (and no row operations) to find x_1 and x_2 that satisfy the equation

$$\begin{pmatrix} 1 & 0 \\ -2 & 1 \\ 3 & -2 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 2 \\ -3 \\ 4 \end{pmatrix}.$$

   (b) Are the vectors $\{\mathbf{v}, \mathbf{w}, \mathbf{z}\}$ linearly independent? Why or why not?

3. *(6 points)* Show that the vectors $\mathbf{p} = \begin{pmatrix} 0 \\ 3 \\ 1 \\ -1 \end{pmatrix}$, $\mathbf{q} = \begin{pmatrix} 6 \\ 0 \\ 5 \\ 1 \end{pmatrix}$, and $\mathbf{r} = \begin{pmatrix} 4 \\ -7 \\ 1 \\ 3 \end{pmatrix}$ are linearly dependent. Express each vector as a linear combination of the other two.

4. *(6 points)* Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be a linear transformation such that T maps $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ into $\begin{pmatrix} 2 \\ 3 \end{pmatrix}$, and maps $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$ into $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$. Find the images of $\begin{pmatrix} 3 \\ -2 \end{pmatrix}$ and $\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$.