

MATH 2100 – HOMEWORK 6

Fall 2022

due Wednesday, **December 7**, at the start of class

Sections 3.3, 4.1, 4.2, 4.3, 4.4

This homework assignment was written in L^AT_EX. You can find the source code on the course website.

Instructions: This assignment is due at the *beginning* of class. Please write the questions in the correct order. Explain all reasoning.

1. Prove that $\mathcal{P}(A \cap B) \subseteq \mathcal{P}(A) \cap \mathcal{P}(B)$.
2. Prove that $(A \cup B) \times C = (A \times C) \cup (B \times C)$.
3. Draw the two-sided arrow diagram for the function $f : \mathcal{P}(\{4, 5, 6\}) \rightarrow \mathcal{P}(\{1, 2, 3\})$ defined by

$$f(S) = \{x - 3 : x \in S\} \setminus \{2\}.$$

4. Prove that the function $h : \mathbb{N} \rightarrow \mathbb{N}$ defined by $h(n) =$ [the sum of the digits in n (in base 10)] is surjective. Prove that it's not injective.
5. Let $c : \mathcal{P}(\{x, y, z\}) \rightarrow \mathcal{P}(\{x, y, z\})$ be the function with the rule $c(A) = \{x, y, z\} \setminus A$, and let $n : \mathcal{P}(\{x, y, z\}) \rightarrow \{0, 1, 2, 3\}$ be the function such that $n(A)$ is the number of elements in the set A . Which composition makes sense, $c \circ n$ or $n \circ c$? For the one that is defined, give the domain, codomain, range, and draw the two-sided arrow diagram.
6. Let $A = \{0, 1, 2, 3\}$ and let $B = \{000, 001, 010, 011, 100, 101, 110, 111\}$ be the set of binary strings with three digits. Define $g : B \rightarrow A$ by $g(s) =$ [the number of 1s in s]. Draw the arrow diagram for the function. Determine whether or not it's injective, surjective, and bijective. Make sure to justify your answers (either with the arrow diagram, or a formal proof).
7. Consider the set $S = \mathcal{P}(\{1, 2, 3, 4\})$. Define $\Sigma(T)$ to be the sum of the elements in T . For example $\Sigma(\{1, 3, 4\}) = 8$. Define the relation $R = \{(A, B) \in S \times S : \Sigma(A) < \Sigma(B)\}$. Answer the following questions.
 - (a) Is R reflexive?
 - (b) Is R irreflexive?
 - (c) Is R symmetric?
 - (d) Is R antisymmetric?
 - (e) Is R transitive?
 - (f) Is R a partial order? If so, draw the Hasse diagram.
8. Let $S = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$ and $R = \{(a, b) \in S \times S : |a - b| = 4\}$. Answer questions (a) – (f) from Question 6.
9. Let $S = [0, 4\pi)$ and define the relation $R = \{(a, b) \in S \times S : \sin(a) = \sin(b)\}$. Answer questions (a) – (f) from Question 6.

10. Let $A = \{1, 4, 7\}$. Give an example of a relation R on A that is
- (a) Transitive and reflexive but not antisymmetric.
 - (b) Antisymmetric and reflexive but not transitive.
 - (c) Antisymmetric and transitive but not reflexive.